IM^{2}C 2022 Australian Judges' Commentary

The Australian IM^{2}C judges congratulate the 93 teams from 25 schools all across Australia who participated in the 2022 Mathematical Modelling Challenge. The two teams chosen to represent Australia at the international round are: *Special-List* from Bray Park State High School in Queensland and *Quality Eggs and Benedict* from Caulfield Grammar School in Victoria.

The 2022 Challenge

The challenge this year was about finding the most efficient method for passengers to embark and disembark an aircraft. Teams of up to four students had five consecutive days to create mathematical models and submit a report of up to 20 pages, a summary and a letter to an airline executive. Any supplementary information, data and code could be added as an optional appendix which was not included in the judging process.

The challenge had several parts to the problem:

1. Create a model or models for boarding/disembarking aircraft with different numbers of entrances, aisles and seats.
2. Use the models to investigate the various effects of stowing carry-on bags, of passengers not following the prescribed methods, and of restricting seating capacity.
3. Compare three given boarding methods, describe two other methods, then recommend, explain and justify a preferred method.

Several features make this an interesting problem:

- Passengers cannot pass each other in the aisles.
- Stowing bags in overhead bins can cause holdups in an aisle.
- Some seated passengers may need to move back into an aisle to allow others in the same row to be seated.
- Every action takes some time to complete but many actions can happen simultaneously throughout the aircraft.
- Each action can depend on previous actions of other passengers in complicated ways.

Finding the best way to interpret, model and solve this problem and report the results was hard because there were many factors to consider and difficult decisions to make in the early stages. Here are some observations and advice from the Australian judges. Several of these are likely to be relevant also to other modelling situations including future entries in IM^{2}C.

Interpreting the problem

There is generally no need to rewrite the problem statement. A condensed interpretation, omitting some of the details, would serve as a good introduction. Teams interpreted the meaning of the practical ranges of boarding times (5th and 95th percentiles) differently, depending on their method of solution. Some generated randomness in the results by varying walking speeds, baggage stowing times or number of bags stowed. Others varied the sequences of passengers in multiple simulations of each boarding method.

Creating models and solving the problem

The big decision in this part of the Challenge was to decide whether to use static equations or dynamic simulations. A few teams were split, trying to do both. Some initial brainstorming with paper and pencil would have revealed the complexity of the interactions between passengers, suggesting the need for simulating multiple passengers simultaneously following simple rules. Those who did not take this reasoning process far enough developed inaccurate models.

Many teams oversimplified the model by multiplying the boarding time of a single passenger by the total number of passengers, for example. Others produced mathematical models of great intricacy.

Not many models used meaningful or memorable variable names, unfortunately. Short or abbreviated names are often much easier to read and understand than single-character or numerically subscripted variables. Parameter values should preferably be derived from measurements. Walking speed, baggage stowage and seat shuffling times can easily be measured in the classroom or in a school bus, which some teams did There is also some literature available on these times, however, some teams did not use the literature to justify their choice of parameter values. Instead they estimated the values with little to no justification. A few used valid statistical distributions of times but with no adequate explanation. Calculating probabilities of seat shuffles was usually done well but directly calculating the probabilities of hold-ups in the aisles was too difficult.

The most successful approaches required some computer programming to simulate the actions of passengers, but only if the programming effort did not overshadow the report. Various languages were used, with *Python* a popular choice. Some tried heroically to use *Scratch*, using 'code blocks', but could not complete the challenge. Most simulations used grid-based finite automata, and a few were frame-based, allowing each action to last a variable number of frames. The programming approach also allowed the same 'model' to be easily applied to different conditions and aircraft designs, and the graphical output was usually quite convincing.

When computer programs are used, care is needed to make sure that readers are given enough information to understand the logic of the program used. It is unwise to assume that readers are familiar with the details of any programming language, and it is generally helpful if variable names are chosen carefully. If external computer programs written by others are used, it is essential that this is adequately referenced and described, and that sufficient information is provided to allow a reader to determine how well you understand their operations.

One relatively simple approach, which was almost used by one team, is the Genetic Algorithm. Boarding sequences could be represented as sequences of simple 'genes', with a 'mutation' exchanging two passengers in the boarding queue. Simulation would be used to determine each outcome.

None of the teams tried to solve the 'opposite' problem of finding the *least* efficient method of boarding. Solving a dual problem can sometimes be easier and give useful insights into the original problem.

Checking and interpreting the results

Comparisons of results as graphical displays of trends or distributions on the same scale were most convincing and self-evident. Separate graphs of distributions of results were much harder to compare and uncaught, unrealistic or inconsistent numerical errors were sometimes found in tabular displays of results. However, teams usually explained and interpreted differences between boarding methods well. It was also important to distinguish between theoretically optimal boarding methods and good practical methods, which many teams did well.

Communication of the methods and results was most effective when the style and structure of the report was neat and consistent, and the narrative clear and simple. The report should tell a convincing story.

General advice

Although this year's problem was perhaps best solved by simulating multiple 'agents' dynamically, we wish to repeat previous advice for solving mathematical modelling problems in general.

1. First spend some time defining the exact requirements of the task and ensure that the entire team understands what each member needs to do. Small misinterpretations at the beginning lead to confusion and can result in time consuming fixes.
2. Allow more than enough time for writing; it takes longer than most people think. Sketch an outline with headings early and create a writing schedule to stay on track.
3. Brainstorm and strategize together. Concentrate on big ideas first. Be clear about what kinds of problems need to be solved and what steps need to be taken. Use sketches, diagrams and short verbal descriptions but avoid mathematical language until there is a clear understanding of what to do.
4. Specify the problems, assumptions and constraints as precisely as possible. Models are approximations of reality, so decide what must be included and what can be ignored. Leave out all that cannot be measured. Explain these decisions clearly in the report.
5. Start developing appropriate mathematical models based on available data only. Start with simple and understandable models. Study what each part does and how the parts combine. Compare different approaches. Use small sets of data to test how the models work and behave so that small changes may be made earlier rather than later. Time is very important.
6. Rather than using fixed, arbitrary parameter values, even if they seem reasonable, try to use parameter values that are determined by the data. For example, try to find models that are functions of known data and can predict values of yet unknown data as well as possible. Such a model justifies itself.
7. Don't be satisfied with the first attempt, but check the robustness of a model against changes in its key assumptions and original parameter estimates.
8. Do not use unnecessarily complex methods to impress the judges. Only attempt more sophisticated methods or models if they can be explained clearly and concisely, and their use is justified by the context. Be very careful about this. Remember that complex models tend to be better than simple models at representing known data but worse at representing new data, which is usually what is required.
9. When satisfied with the models and methods, decide how to communicate with the readers. Introduce ideas in a logical order. Introduce all variables clearly. Choose variable names carefully so that they remind a reader of their meaning. Do subscripts of sequence variables make sense? Do they start from zero or from one? Be consistent. Don't switch notation between sections of the report. Number equations if they are referred to in the text.
10. Explain how methods work using simple language, simple diagrams or simple examples so that the readers can follow with the least amount of effort. Ask yourself, using your descriptions, would another person be able to implement the methods and get the same results?
11. Interpret the results. Do they make sense? Have they answered the questions? What do they mean? How do the methods and models compare to others? Can they or should they be changed or improved? If so, how or why? Think carefully.
12. Check and edit the final report as a whole. Use the same formatting for all sections. Avoid repeating the same ideas in different sections. List the relevant and accessible source material.
13. Place only non-essential material such as data tables and computer programming code in the appendix. Judges may sometimes refer to this material but do not consider it in the judging process.

In summary, we congratulate all participating teams on their efforts during this challenge, we thank each school for their support and we hope every team member will continue to improve their skills in thinking and communicating mathematically. The originality of the models developed was inspiring, and we look forward to seeing the entries to the 2023 Challenge.