

Example problem

Level: Junior or middle secondary

Köchel numbers



Photo credit: Portrait of Mozart by Barbara Krafft

Describe the real-world problem

'It is sobering to realise that when Mozart was my age he had already been dead for three years.'

— Tom Lehrer, mathematician/comedian

Wolfgang Amadeus Mozart, one of the most influential composers of the Classical era, was born January 27, 1756, in Salzburg, and died December 5, 1793, in Vienna. He composed more than 600 works over his career.

Ludwig von Köchel, Viennese botanist, mineralogist and educator, published an inclusive, chronological catalogue of Mozart's work in 1862.

Köchel (K) numbers are assigned sequentially according to the date of composition. For example, Mozart's opera *The Magic Flute* is given the Köchel number 620, and is (approximately) the 620th piece of music Mozart composed.

Compositions completed at the same time are listed K69, K69a, and so on.

Specify the mathematical problem

A new composition by Mozart completed in April 1784 has come to light. What Köchel number should it be given?

Formulate the mathematical model

Data

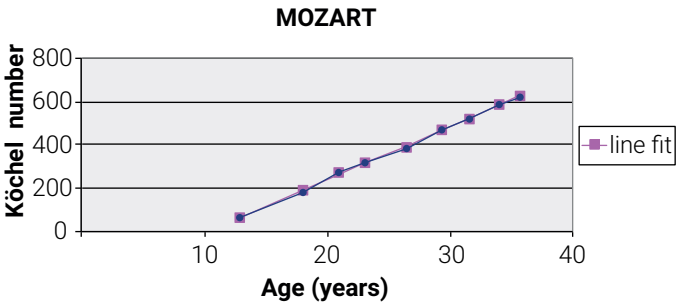
Given the information in the table, find a way of linking Köchel numbers with the dates of completion.

K	Date completed	Composition
65	Jan 1769	Dance music
176	Dec 1773	Dance music
271	Jan 1777	Piano concerto
317	Mar 1779	Mass
385	July 1782	Symphony: <i>Haffner</i>
470	Apr 1785	Andante for strings
525	Aug 1787	Serenade: <i>Eine Kleine Nachtmusik</i>
588	Jan 1790	Opera: <i>Così fan tutte</i>
620	Sept 1791	Opera: <i>Die Zauberflöte</i>

From the table above, we translate the completion dates to Mozart’s age in years, knowing his birth date. (K65 corresponds to 13.0 and K620 corresponds to 35.7 etc.) These can be added as another column beside ‘date completed’ in the table. Plotting the points on a grid with Mozart’s age along the horizontal axis (x), and Köchel numbers on the vertical axis (y), they are seen to lie approximately along a straight line.

Placing a ruler (by eye) through the points identifies a line which in our judgment best fits the set of points. The line can then be drawn – a so-called ‘line of best fit’. The equation to this line will give a link between age and Köchel number.

There are several ways of finding this equation. For example, if our ruled line passes through the endpoints (13.0, 65) and (35.7, 620) of the graph then its equation will be $Y = 24.4x - 252.8$ (working to one decimal place).



Solve the mathematics

The date of the new composition (April 1784) corresponds to $x = 28.3$

Substituting in $y = 24.4x - 252.8$ gives $y = 436.5$.

Interpret the solution

An appropriate Köchel number would be $K = 437$ or K437a or K437b.

Evaluate the model

Referring to the table, a composition with $K = 437$ (April 1784) should lie towards the upper end of the interval between $K = 385$ (July 1782) and $K = 470$ (April 1785). It does, so it is reasonable to infer that the model is suited to its purpose.

On the other hand, the model was derived using only nine points. It would be reasonable to repeat the calculations using research to locate say 20 points to use as a starting basis, and to note and comment on any differences.

Refinement using technology

Students familiar with graphical calculator technology will likely identify the opportunity to use the regression facility to obtain the line of best fit by technical means. This is of course a legitimate approach, but one which should not be forced on those who are not familiar with the appropriate technology. (Diversion down unfamiliar technological paths has been shown to impede progress within modelling problems.)

In the present case, the application of the linear regression facility is shown in the straight line drawn on the graph above. Its equation is $y = 24.8x - 259.3$.

This leads to a value of $K = 443$ which is within 1.5% of the value calculated using eye and hand. The final choice would involve searching out and examining the Köchel numbers already assigned to compositions around the date in question.

Report the solution

The report should contain the foregoing, perhaps with a summary statement that generalises the usefulness of the model beyond the single case which is featured here. We can note also that a formula such as this can be used generally to check for errors in assigning Köchel numbers, by looking for wayward results when the formula is applied to large numbers of data.

There should also be comment on reservations concerning precision, on account of approximations used in the approach.