Example problem

Level: Junior or middle secondary

(IM²C international mathematical modeling challenge)

Waste not, want not

Describe the real-world problem

On 16 February 2016, the Australian population reached 24 million people. Waste generation rates are a function of population growth, the level of urbanisation and per capita income and Australians now produce about 50 million tonnes of waste each year, averaging over 2 tonnes per person. There are more of us and we generate more waste per person, each year.

In the period 1996–2015 our population rose by 28% but waste generation increased by 170%. Waste is growing at a compound growth rate of 7.8% /Year.

Source: Mike Ritchie – Director, MRA Consulting Group https://blog.mraconsulting.com.au/2016/04/20/state-of-waste-2016-current-and-future-australian-trends/

Australia is one of the highest waste producers in the world, recently ranked in the top five waste producing nations, on a per person basis. In a year, we produce the equivalent of three million garbage trucks full of compacted rubbish. Every year the average Australian family produces enough rubbish to fill a three-bedroom house from floor to ceiling.

Some waste we are directly responsible for (e.g. household waste); other waste is generated on our behalf (e.g. from manufacturing processes, the building industry, road construction etc.). It is convenient to describe total waste generation in terms of amount per person. From the figures quoted, the total waste produced annually is currently 2.1 tonnes per head of population (approximately). Of this, household waste accounts for about one-seventh of the total.

On the positive side, recycling is growing at a faster rate and for the first time since 2005 we have seen a decline in tonnages of waste sent to landfill (in the most progressive states). We now recycle approximately 58% of all the waste we generate and landfill the rest.

Several modelling problems are suggested by this background information, in terms of implications for the future. We consider one such problem.

Specify the mathematical problem

Find estimates in kilograms (or tonnes) for the total amount of waste produced in Australia over the next century.

Formulate the mathematical model

Assumptions

- Total waste produced in a year = population × waste ÷ person.
- Rates of increase that have applied over the past 20 years will continue into the future.
- Recycling or use of landfill do not enter at this point, as they are about managing waste that has been already produced. Here we are concerned with the production of total waste, to which re-cycling or landfill might later be applied.

Parameter values

- Total waste is growing at a compound rate of 7.8% per year.
- Initial (2016) value for population is 24 000 000.
- Initial (2016) amount of waste per person is 2.1 tonnes per year.
- Estimate of amount of total waste produced in 2016
  = 24 000 000 × 2.1
  = 50 400 000 tonnes (assumption 1)

Model development

From the descriptions provided on the MRA website, rate of growth of waste follows a compound interest pattern.

Principle of compound growth

Amount next year (A_1) = amount this year (A_0) + added amount (interest).

$A_1 = A_0 + rA_0 = A_0\,(1+r)$ where r = annual compounding (interest) rate.

Then $A_2 = A_1 + rA_1 = A_1\,(1+r)$ gives the amount for the second year, and so on year by year.

Applying this principle to total waste generation (W):

$W_1 = W_0\,(1+w)$

$W_2 = W_1\,(1+w)$ and so on

Where W_0 =50 400 000; w =0.078.

Applying assumption (2) we now extend predictions into the future.

Solve the mathematics

This is readily achieved using a spreadsheet.

	A	B	C	D	E
1	Initial values	Waste growth rate	year	waste/year (tonne/yr)	Total waste (tonne)
2	Population (persons)		1	= A3×A6	=D2
3	24000000		2	= D2×(1+B6)	= (E2+D3)
4			3	Copy	Copy
5	Waste/ person/yr	Waste/yr	4		
6	2.1	0.078	5		
"			"		
"			"		
101			100	8.5445E+10	1.1803E+12

Interpret the solution

Column A contains the initial values of population, and of total waste produced per person in 2016.

Column B contains the assumed annual rate of increase of waste.

Column C contains the consecutive years over which the calculation runs.

Column D contains the total waste (in tonnes) produced on a yearly basis since 2016.

Column E contains the accumulated waste produced since 2016.

The last entry in column E estimates the number of tonnes of waste produced over the century – it represents an answer to the question originally posed.

Evaluate the model

Writing out the estimate for total waste produced over the century looks like this:

From column E: Total waste produced over 100 years = 1 180 300 000 000 (over 1 trillion tonnes!)

This looks 'rather high' in practical terms, but we need something closer to home to relate to.

Column D tells us the amount of total waste generated on a yearly basis will be 85 440 000 000 tonnes per year in 100 years' time.

The background information says that over a 20-year period the population grew by 28%.

Assuming that population continues to grow at about the same rate it will not be too much in error to assume that in 100 years it will have grown to around 70 million.[7]

Dividing 85 440 000 000 by 70 million gives 1220 tonnes produced per person (approx) in the year 2115.

Noting that household waste formed about 1/7 of total waste, and assuming the proportion remains stable, this gives a quantity of about 174 tonnes of household rubbish per person for the year 2115.

On average this amounts to the production of about 0.48 tonnes (480 kg) per day for each person. That sounds like a lot of garbage (in more ways than one!)

After checking the mathematics, we are sceptical about the model predictions. What should be revisited?

[7] It isn't too much trouble to infer a compounding growth rate for population of 1.112 % p.a. from the given data, which when projected over 100 years from 2016 gives a population of about 73 million

Model refinements

Assuming that the initial values are reliable, and population growth rates are fairly stable, we can identify growth rate for waste production 'w' as speculative. On the one hand we have assumed that the scale of growth observed over a 20-year period will continue for the next century. Is this reasonable? But beyond this, a closer inspection of the indicated growth rate (7.8% per year) given on the website is not consistent with the associated figure of a 170% increase in waste over the 20-year period – the implied rate is much less (about 3.95%).[8]

We can run the spreadsheet multiple times, using different values for 'w'.

Try halving the rate of growth of waste production. How does that look? What would it mean in real-life terms? Is it manageable? Similarly, for other choices.

Alternatively, set what are believed to be manageable levels of waste production and adjust 'w' in the spreadsheet to achieve these outcomes. This gives an idea of what efforts need to be made to contain the waste problem.

Additional problem contexts to explore

A. Recycling

How does Australia's recycling compare to the rest of the world?

Australians compete against the rest of the world in many ways and recycling is no exception. In some areas, such as newspaper recycling we have been world leaders for years, but we need to catch up in others. According to the MRA consulting website we now recycle 58% of all waste.

What impact would this have on implications from the previous model? Make some assumptions about future improvements in recycling, and follow their implications by including them in a new model.

B. Implications for landfill

www.pc.gov.au/__data/assets/pdf_file/0016/21904/sub028.pdf

Australia has a strong dependence on landfill as a form of waste management, since the majority of waste that is not recycled or re-used in Australia is compacted and disposed of in landfills. The website contains much information about land filling including the following:

'… the depth of the landfill is assumed as 20m, the waste density is assumed as 750kg/m3 and the capacity is assumed to be 3 million tonnes.'

Build a model that incorporates landfill as a means of disposing of waste that cannot be re-used or recycled, and consider future implications of the outcomes it produces.

[8] This example shows the importance of evaluating models in terms of their real-world implications. In this case, evaluation led to the identification of an inconsistency in the data. In general, evaluating model outcomes means that the accuracy of claims made in the media can often be tested, which is another reason why the ability to do mathematical modelling is important for informed citizenry.