

Example problem

Level: Middle secondary

Intermediate modelling

How many trees make a newspaper?



Describe the real-world problem

How many trees go into your morning read?

According to the environmental conservation organisation Clean Up Australia, Australians' per capita consumption of paper products is high, and increasing every year. Australians consume more than four million tonnes of

paper and cardboard annually. The environmental impact of paper consumption is significant. Although most used paper could be recycled, around half of it ends up in landfill.

Specify the mathematical problem

Estimate how many trees are needed to provide a year's circulation of a major newspaper.

If the task is planned ahead of time, students can be asked to track daily papers for a week (or several weeks), and to bring newspapers to class when the problem is set.

Formulate the mathematical model

This is an ideal team task. Form teams of four students who can then brainstorm, within their team, for a few minutes, a list of the data they will need to create a model, and the assumptions they will need to make. Outcomes can then be discussed and debated by sharing with the whole group. (Typically, not all necessary data and assumptions will be recognised at the start.)

Some examples are given below.

Identifying data required

Data required include:

- how much paper is in a newspaper, possibly considering
 - dimensions of a page
 - margin data, if it is decided to find how much paper is unused for actual print
 - daily page count or average pages over a week
 - weight of a page of newsprint
- how many newspapers are produced in a year, i.e., circulation (number of copies printed)
- weight of newsprint paper obtained from an average tree used for pulp; i.e., the number of trees required to produce a given weight of paper.

An ice-breaking very simple team activity is for each member to measure the dimensions of a page, both total and associated with print area. Averaging will give a measure to use in the model.

Collecting data

How this proceeds depends on how much time is allotted to the task – for example spread over a week, or allotted a limited amount of class time? If the former, after identification of needed information, student teams can be assigned tasks of collecting their required data, such as:

- conducting internet research to find circulation figures of publications
- conducting internet research to identify relevant ways to convert quantities of paper into weight (kg). (If an electronic balance is available, weighing some newspaper is easiest)
- conducting internet research to estimate the amount of paper provided by a tree.

If time is limited, then websites can be provided to students. In that situation, the web links can be withheld until the students have identified the need for the associated information.

Many of the data requirements and ways of finding them will generate discussion. Weight of paper, for example, is a complicated measure. An internet search is likely to return some information about paper weight measured in standard base weight, using pounds (lb). A search will also find, however, that in many countries using the metric system, the relevant measure is grammage (grams per square metre, or g/m²).

In Australia, the weight of paper and paperboard is most commonly expressed as grammage, and it will be discovered that since the 1970s, the grammage of newsprint has decreased from a global standard of 52 g/m² to 48.8, 45 and 40 g/m². This is cheaper for newspaper producers, and increases the viability of forest resources. Students can consider why publishers might choose a certain weight of newsprint, and experiment with various weights in their model.

Additionally, there is no standard agreement on how many trees are required to create one paper’s worth (or one pound, one kilogram, one tonne) of newsprint. Hence estimates are needed and these can vary with the source, because of variations in production processes.

The following websites are useful references:

Conservatree
<http://conservatree.org/learn/EnviroIssues/TreeStats.shtml>

Australian Science
<http://www.australianscience.com.au/environmental-science/paper-consumption-impact-in-australia/>

Situational assumptions

- The number of pages is the same each week day, and each weekend, throughout the year.
- The pages have standard dimensions.
- The paper quality (thickness, weight) remains constant.
- The model excludes inserts and magazines, and assumes a 52 week year and additional copies not sold.

Solve the mathematics

Example data

For the purpose of this resource, an example data set is given below for a fictitious daily regional newspaper.

Given this information and the assumptions, weekly calculations can be completed and scaled up to give yearly estimates.

Calculate paper usage

Per week
Single pages = (78 × 153 763 × 5) + (96 × 199 153)
+ (96 × 359 088) = 113 558 706
Single newsprint sheets (2 pages use 1 sheet of paper)
= 0.5 × pages = 56 779 353

Dimensions	
Width (cm)	29
Height (cm)	40
Top margin (cm)	1.8
Bottom margin (cm)	1.6
Left margin (cm)	1.8
Right margin (cm)	1.8
Average number of pages	
Monday to Friday	78
Saturday and Sunday	96
Circulation	
Monday to Friday	153 763
Saturday	199 153
Sunday	359 088
Weight	
Assume a representative standard newsprint grammage	45 g/m²
Number of trees to weight of paper	
Internet research informs assumption that one tonne of paper consumes approximately 12 full-grown trees (Conservatree)	12 trees per tonne

Per year

Single newsprint sheets = $52 \times 56\,779\,353 = 2\,952\,526\,356$

Area of sheet = $29 \times 40 = 1160 \text{ sq cm} = 0.116 \text{ sq m}$

Total area = $0.116 \times 2\,952\,526\,356 = 342\,493\,057 \text{ sq m}$

Convert area of newsprint to weight

For this example, assume a representative value for grammage 45 g/m^2 . Other assumptions can be adopted to vary the calculations.

Standard newsprint grammage = $45 \text{ g/m}^2 = 0.045 \text{ kg/m}^2$

Weight of newsprint = $0.045 \times 342\,493\,057$

= $15\,412\,187.6 \text{ kg} = 15\,412.2 \text{ tonne}$

Convert weight of newsprint to trees

For this example, we assume a representative 12 trees per tonne of paper. Other assumptions can be adopted to vary the calculations.

1 tonne of newsprint uses 12 trees

$15\,412.2 \times 12 = 184\,946 \text{ trees}$

Estimate area of trees needed

It is of interest to consider what this might look like in terms of forest area.

Internet research can be used to discover how many trees on average can be harvested/planted per hectare. This may result in different possible calculations.

State-based government departments of primary industries, for example, provide information on plantation management.

As an example calculation for this problem, using an average value of 1000 stems per hectare, 184 946 trees are needed to produce a year's worth of the example newspaper; that is, 184.9 hectares or 1.85 sq km.

Generalise solution

Define symbols: page dimensions (l, w); number of pages weekly (n); weekly circulation (c); grammage (g); trees/tonne of newsprint (t).

Create a formula that can be used to estimate the number of trees needed for the annual production of any daily newspaper.

Sensitivity testing

Using the formula for convenience, it is useful to vary the inputs from their values used in the example calculation and see how the outcome is affected. This tests the sensitivity of the result to changes in the inputs, and gives a sense of the robustness of estimates.

Interpret the solution

In this problem, the building up of the solution has involved a consistent linking of all mathematical measures with their real-world meanings. So interpretation has followed mathematical calculations at all points.

Evaluate the model

Firstly, the calculations should be checked to see that all essential variables have been built into the solution, and that arithmetic has been conducted accurately – noting that conversion between different systems of units may be involved. The generalisability of the model can be evaluated in terms of the formula developed for number of trees used, and its usefulness also in terms of insights gained from sensitivity testing.

Refinement

The model can be refined by considering the impact of recycling. As an example for this problem, we have used information about recycling from Clean Up Australia (http://www.cleanup.org.au/files/clean_up_australia_paper_cardboard_factsheet.pdf).

Australia leads the world in newsprint recycling

Australia's newsprint recycling rate is almost 10 per cent higher than in Europe, according to figures from the Publishers National Environment Bureau and media advocacy group The Newspaper Works. Annual figures over more than a decade show that Australia consistently recovers around 78 per cent of all newsprint. The average recycling rate for newsprint in Europe is 69 per cent. Australian newspapers only use tree pulp from plantation pine and use up to 40 per cent recycled materials; but newspaper and magazines are the most abundant forms of paper waste.

How many trees are saved annually by using 40 per cent recycled materials in producing new newsprint, for the newspaper used in this example question?

$0.40 \times 184\,946 = 74\,000$ (approximately)

Report the solution

The modelling report could contain all the components of the modelling problem and its solution, as developed in preceding sections. The report should synthesise this data into a cohesive narrative, considering the implications of Australian plantation processes and use of recycled materials in producing new paper products.