Example problem

Level: Senior secondary

Harder modelling

Temporary traffic lights

Describe the real-world problem

Lane closure between Newtown and Highbury

1 February, 2015. Traffic restrictions will apply on the Kings Highway from 3 March, to allow for the construction of a new eastbound overtaking lane approximately one kilometre east of Highbury. To facilitate this work, the existing eastbound traffic lane will be temporarily closed for approximately three weeks. For safety reasons, traffic will be restricted to one lane, on a 600-metre section of the Kings Highway, through the work site where 55- and 25-kilometre-per-hour speed limits will be in place. Temporary traffic signals will be in use to manage traffic flow and delays can be expected.

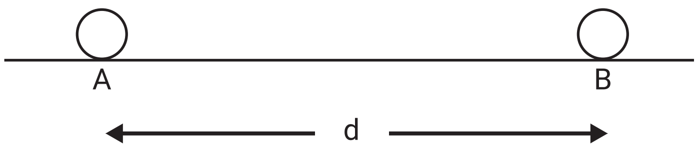
Specify the mathematical problem

Two-way traffic along a 600m single lane section of roadway is controlled by temporary traffic lights at each end. A speed limit of 55 km/h applies. How should the timing of the lights be set to achieve an efficient flow of traffic in both directions?

Formulate the mathematical model

We first need to understand how the light cycle operates.

- Traffic lights are at A and B, where d = 600, and V (max speed) = 55km/h.
- Assume that the road is a main highway – no pushbikes or farm machinery allowed.
- Assume that the traffic is similar in both directions and sufficiently dense to build up while the light is red.
- Last car through on a green light at A will take (d/V + safety margin(s)) sec to clear the road, before the light at B turns green
- Let T (same for both) be the green light time at A and B.



Complete cycle of light changes at A (same for B)					
Time	0	T	T + d/V + s	2T + d/V + s	2T + 2d/V +2s
Light at A	G	R	R	R	G
Light at B	R	R	G	R	R

Solve the mathematics

The length of cycle is $2(T + d/V + s)$
so, the proportion of green light time = $T/2(T + d/V + 2s)$.

Using $V = 15$ m/s (54km/r) and and a safety margin of 5s gives proportion of green light time = $T/2(T+45)$.

For example: $T = 60$ gives a value of 0.29 (29%)
$T= 180$ gives a value of 0.4 (40%)

Interpret the solution

Does this mean that longer green light times (at both ends) are more efficient?
We don't know.

What are the implications for traffic flow?
Our model so far is not wrong, but we evaluate it as inadequate to answer this question, as yet we have no basis for assigning values to T.

How do we obtain estimates of the respective numbers of vehicles arriving at the lights, and leaving while they are green?

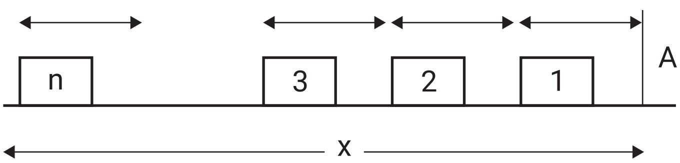
First, consider vehicles arriving at A during a cycle of light changes.

Defensive driving advice says to leave 2s between vehicles passing a roadside marker.

On a busy approach road this gives an estimate of the rate at which vehicles arrive, and typically join a waiting queue. Keeping things general, we denote this parameter by b.

So, the number of vehicles arriving during a cycle of length $2(T + d/V + s)$ is given by
$N = 2(T + d/V + s)/b$
that is $N = 2(T + 45)/b$

A model for vehicles leaving A during green light time is shown below.



Next, consider vehicles waiting for green light.

Average vehicle length (including space to vehicle in front) = l.
Vehicle n is last through the light when it turns green.
a is the average delay time between successive vehicles taking off.
So available time in motion for nth vehicle to reach lights = $T - an$.

Assume that we can ignore short period of acceleration to speed V.
Then $ln = V(T - an)$ and so $n = VT/(l + aV) = 15T/(6 + 15a)$.

For cars to just clear during a cycle of light change we need $n = N$.

Thus $15T/(6 + 15a) = 2(T + 45)/b$
So $T = 90(2 + 5a)/(5b - 10a - 4)$

We have noted above a reasonable (minimum) estimate for b: $b \approx 2$.

Drivers are 'rearing to go' when vehicle in front moves so estimate $a \approx 0.5$.

This gives $T \approx 405$.

Evaluate the model

Nobody is going to sit comfortably for 6 to 7 minutes in a queue with no sign of activity; so, review the outcome.

Do we need a new model or to amend some aspects of the approach?

Suppose $T = 240$ (4 min): $n = 267$ and $N = 285$ (18 vehicles wait a turn)
$T = 180$ (3 min): $n = 200$ and $N = 225$ (25 vehicles wait a turn)
$T = 60$ (1 min): $n = 67$ and $N = 105$ (38 vehicles wait a turn)

But now the percentage missing out is approaching 40 per cent, and the build-up will be rapid. So balance driver frustration with volume of traffic movement. How many light changes are tolerable on a busy road: Two? Three? Four?

Conduct a retrospective estimate of omitting acceleration phase: If $T = 180$, the nth vehicle (number 200) will have travelled $200 \times 6 = 1200$ metres when it just makes the light, and will have been travelling for about 80 s. The initial phase is a very small fraction of this.

Revisit parameter values: l, s, a, b.

Note from the formula for T that we must have: $5b - 10a > 4$.

Consider different times of day: the effect of b.

Consider the influence of heavy truck presence.

Investigate the effect of including an acceleration phase, for example:
$T = 240$: $n = 263$ versus 267 (change 1.4%)
$T = 180$: $n = 197$ versus 200 (change 1.5%)
$T = 60$: $n = 63$ versus 67 (change 5%)

Report the solution

The modelling report should contain all the above components of the modelling problem and its solution. The report should synthesise this data into a cohesive narrative, considering the implications for safety and driver frustration. The report could recommend a particular option that best balances the factors of wait time and traffic build up.

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