Example problem

Level: Senior secondary

Harder modelling

(IM²C international mathematical modeling challenge)

Super Size Me

Disclaimer: This problem is about lifestyle choices. It explores the impact of energy intake (food selection) and exercise on bodyweight, and the particular source of food products is irrelevant.

Describe the real-world problem

Australian obesity a big problem

29 May, 2014. Worldwide adult obesity rates have jumped nearly 30 per cent in the last 30 years, according to new analysis from the Global Burden of Disease Study 2013, published in medical journal *The Lancet*. Adult obesity rates in Australia are climbing faster than anywhere else in the world, and are only slightly less than those of the United States. In Australia, 63 per cent of adults are overweight or obese. In response to the study, health experts have called on the governments in Australia to commit to a national strategy to address overweight and obesity.

Commenting on the implications of the study, Professor Klim McPherson from Oxford University in the UK stated that an appropriate global response to the worldwide obesity rates would focus on 'curtailing many aspects of production and marketing for food industries'.

This kind of public concern is not new. In America in 2004, concern over a perceived link between obesity and food industries led to the making of the documentary film *Super Size Me* (2004). For 30 consecutive days, the film's creator Morgan Spurlock Morgan ate three meals daily consisting of nothing but McDonald's food and beverages (consuming approximately 5000 calories daily). If offered an 'upsize' he always took it, and limited his daily exercise to that of the average American office worker. He ate everything on the McDonald's menu at least once. Spurlock, a 32-year-old male who was 188 cm tall and weighed 84.1 kg at the start of the experiment, documented as his weight increased by 11.1 kg. There were other damaging effects to his body.

Specify the mathematical problem

Develop and evaluate a mathematical model to describe the weight gain experienced by Morgan Spurlock. Use the model to explore the respective effects of calorie intake and exercise. You may find useful information on energy and activity levels in the appendix at the end of this question.

Formulate the mathematical model

Data

- The unit of energy is the kilocalorie (Calorie) where 1 Calorie = 4.18 kilojoules.
- 7700 kcal ~1kg (biological data).
- The average energy intake per day, I, is 5000 Calories (from film description).

Assumptions

- Energy processing is continuous, with no delays in converting Calories into body functioning, and excess Calories to additional weight. That is, individual daily effects are averaged out over periods of time.
- Weight change (per day) is determined by energy intake (per day) less energy used (per day).
- Energy used per day = the rate at which a body burns energy, that is, the basal metabolic rate (BMR) + energy used in activity (conduct web search to collect data, or see the appendix to this resource).

Basal metabolic rate

Web search identifies the Mifflin formula, circa 1990, specifying that: BMR (Calories) = 10w + 6.25h – 5a + 5 for males, where w is weight in kilograms, h is height in centimetres and a is age in years.(For females BMR = 10w + 6.25h – 5a – 161.)

Since the experiment lasts 30 days, we assume that age and height can be treated as constants. The film description specifies that a = 32, h = 188, and starting weight = 84.1 kg.
Hence take BMR = 10 × weight + 1020

Energy used in activity

From the film description we assume a sedentary lifestyle over the course of the experiment, and from the information in the appendix this implies an additional energy usage per day of about 0.2 × BMR.
Hence, energy used per day is 1.2 BMR = 12w + 1224.

Converting energy to weight

Weight today = weight yesterday + (Intake in calories yesterday – calories used yesterday) converted to weight.

Let w_n be weight after day 'n' (on day 0 original weight is w_0 = 84.1 kg).
I = average daily intake of Calories (I = 5000 from film data).
Let E_n be the energy used on day 'n' (E_n = 12w + 1224).

$w_1 = w_0 + (I - E_0)/7700$
$w_1 = w_0 + (3776 - 12w_0)/7700 = 3776/7700 + (1 - 12/7700)w_0 =$
$0.4904 + 0.9984\ w_0$

Hence $w_1 = a + bw_0$ where a = 0.4904, and b = 0.9984.

Similarly $w_2 = a + bw_1$ and so on… $w_{30} = a + bw_{29}$.

Solve the mathematics

There are a number of methods that could be used to solve the mathematics.

Calculator

w_1 = 0.4904 + 0.9984 x 84.1 = 84.66
w_2 = 0.4904 + 0.9984 x 84.46 = 84.82
and so on.

But this is tedious.

Spreadsheet

See table below.

Row 3 contains the initial values (day zero).

Formula for BMR is in column E. For example, E4 = (10*B4+6.25*D3-5*C3 + 5)

Formula for energy used in activity is in column G. For example, G4 = (F3*E4)

Formula for weight is in column B. For example, B4 = (H3 – E3 – G3)/7700 + B3

Geometric series

$w_1 = a + bw_0$

$w_2 = a + bw_1 = a + b(a + bw_0) = a(1 + b) + b^2\ w_0$

$w_3 = a + bw_2 = a + b(a + bw_0) = a(1 + b + b^2) + b^3\ w_0$ … leading to:

$w_{30} = a(1 + b + b^2 + \ldots + b^{29}) + b^{30}w_0 = a(1 - b^{30})/(1 - b) + b^{30}w_0$

w_{30} = 94.53.

Calculus

If we treat time as continuous then we can approach a solution through calculus.

Let weight at time t (days) after the start of the 'diet' be w (kg) and consider the change in weight from w to w + δw that occurs between t and t + δt.

If the average daily energy intake is I, then during time interval from t to t + δt intake is ≈ Iδt (Calories).

Similarly, the energy used between t and t + δt ≈ 12w + 1224 (Calories).

Hence δw ≈ [I – (12w +1224)]) δt/7700 so that in the limit dw/dt = (3776 – 12w)/7800.

Thus,

$$\int \frac{dw}{(3776 - 12w)} = \int \frac{dt}{7700}$$

	A	B	C	D	E	F	G	H
1	day	weight	age	height	BMR	Activity factor	Activity energy	Energy Intake (I)
2		(kg)	(yr)	(cm)	(kcal/day)	(BMR multiplier)	(kcal/day)	(kcal/day)
3	0	84.10	32	188	1861	0.2	367	5000
4	1	84.46			1865		368	5000
5	2	copy	—	—	copy	—	copy	copy
33	30	94.51			1965		393	5000

We then have $\ln(3776 - 12w) = -0.00156t + c$.

Hence $(3776 - 12w) = Ae^{-0.00156t}$ (where $A = e^c$).

When $t = 0$, $w = 84.1$ so $A = 2767$ and
$w = (3776 - 2767\, e^{-0.00156t})/12$

When $t = 30$, $w = 94.63$

The latter approach would be unlikely to be within the scope of Year 10 students, for whom a spreadsheet solution would be accessible. In general, of course, if a problem can be approached in more than one way then initial insights are confirmed or challenged, and sometimes new insights obtained.

Interpret the solution

Before leaping to interpretation it pays to reflect back on the context of the modelling. Here a condition was that Morgan consumed approximately 5000 Calories each day. So in reflecting on our solution it would be reasonable to allow the average amount to vary say by 5 per cent from this figure; that is, to carry out calculations assuming average intakes of between 4800 and 5200 calories. What does considering a range of values tell us?

We find that for I = 4800, the predicted weight gain is 9.7 kg (approximately), while for I = 5200 the predicted gain is 11.2 kg (approximately).

Activity level is a more tenuous measure: allowing a 20 per cent tolerance above (0.24) and below (0.16) the value used, with I = 5000 gives a range of weight after 30 days of between 94.2 and 94.8 kg.

It is reasonable to say that the model, built on the basis of daily energy intake (food) and daily energy use (BMR + additional activity) tracks the increase in weight in a logical and coherent way. Given the assumptions it is this property, rather than precise numerical values, that are important for evaluation purposes. (The calculated weight gain for the standard conditions is about 6 per cent lower that the reported figure.)

Evaluate the model

We should not think of this modelling exercise primarily as an attempt to predict precisely the weight gain of Morgan Spurlock over 30 days. Its purpose, rather, is to develop a model that is applicable in estimating the effects of energy intake and energy use on body weight. The film provides specific data against which to test the model.

Given that it appears sensitive to the 'right' data, and gives results that are consistent with the outcomes reported in the film, we now consider what further insights might be obtained. We can consider some hypothetical circumstances that give insight into the respective impact of food intake and exercise. (This represents a move from the evaluation phase back into modelling activity.)

Suppose we hold energy intake at 5000 and double (increase by 100 per cent) the activity level from 0.2 BMR to 0.4 BMR, assuming that such a diet is compatible with such good intentions. Then we obtain a weight after 30 days of 93.1kg compared with 94. 5. A modest reduction for hard work!

Suppose instead we reduce the daily Calorie intake to 4000 (20 per cent reduction) and the original sedentary lifestyle is maintained. Now after 30 days we obtain a weight value of 90.75 kg, almost 2.5 kg less than the previous value.

Other similar calculations that represent lifestyle decisions drive home the information that diet is the predominant problem, and exercise can only go so far if that is not addressed.

Refinement of the model could be achieved through conducting another scenario.

Sixteen-year-old Sarah weighs a steady 55 kg and had an activity level that is 'lightly active'. She receives a skateboard for her birthday and now skateboards for an hour a day, adjusting her food intake so that her weight remains at 55 kg. Recently some fast food outlets opened near her home, and after skateboarding Sarah now has a daily routine in which she eats a cheeseburger and drinks a 375 ml coke. The rest of her diet remains the same. What is the effect of this habit on Sarah's weight after two months (60 days)?

(Alternatively, replace Sarah's information with a personal choice of weight, food and exercise).

Further information about food content and exercise is provided in the appendix.

Report the solution

After investigating these problems, write a letter to yourself summarising the understanding you have obtained about the effects of food and physical activity on fitness. Include advice for maintaining a healthy lifestyle.

The letter should summarise briefly the main findings from the two problems, based on Morgan and Sarah. It should highlight insights obtained about the impacts of exercise and diet, and perhaps contain self-directed advice for a healthy lifestyle.

Appendix

Basal metabolic rate (BMR) is the rate at which energy is used when the body is at complete rest. Use an internet search to find a way of estimating BMR (Mifflin formula).

To find the total energy used it is necessary to estimate the contribution of additional physical activity. There are various ways to do this but a useful one is to express it as a multiple of BMR, which varies with lifestyle.

Examples might include:

- sedentary, little or no exercise, desk job = BMR × 1.2
- lightly active, exercise or sports one to three days per week = BMR × 1.375
- moderately active, exercise or sports six days per week = BMR × 1
- very active, hard exercise every day, or exercising twice per day = BMR × 1.725
- extra active, hard exercise two or more times per day, or training for marathon or other intensive event = BMR × 1.9

When more energy (calories) are taken in by way of food than are needed by the body, the excess calories are converted to extra kilograms of weight at a rate of 7700 Calories = 1 kg.

Activity	Energy used (kcal/kg/h)
Sitting quietly	0.4
Writing	0.4
Standing relaxed	0.5
Driving a car	0.9
Vacuuming	2.7
Walking rapidly	3.4
Skateboarding	5.0
Running	7.0
Tennis	7.0
Swimming	7.9

Sample fast food items	Energy (kcal)
Plain hamburger: ground beef patty, grilled (40 g), hamburger bun (65 g) and salad	328
Plain hamburger and medium chips: ground beef patty, grilled (40 g), hamburger bun (65 g) and salad	497
Cheeseburger: ground beef patty, grilled (40 g), hamburger bun (65 g), lettuce, sliced tomato and onion, tomato sauce, cheese (16 g)	391
Cheeseburger and large chips: ground beef patty, grilled (40 g), hamburger bun (65 g) and salad, large portion of chips	728
Double hamburger and medium chips: 2 ground beef patties, grilled (80 g), hamburger bun (65 g) and salad, medium portion chips	606
Pizza with cheese, tomato and olives: medium portion (90 g)	223
Pizza with cheese, tomato and olives: large portion (340 g)	844
Lemonade (375 ml)	158
Dry ginger (375 ml)	124
Bundaberg ginger beer (375 ml)	188
Coca Cola (375 ml)	158
Pepsi cola (375 ml)	169
Fanta (375 ml)	210