Howzat!

Courtesy of *Teaching Mathematics and its Applications*

Describe the real-world problem

The picture above was taken during a cricket Test Match between Australia and England at the Melbourne Cricket Ground. The photo shows the English batsman Colin Cowdrey trying to make his ground while taking a sharp single, against a run out attempt with Australian Wally Grout over the stumps. At the time there was no third umpire so the players had to rely on the square leg umpire for the decision. An umpire's judgment is still called on in all forms of cricket, except at the highest level where technology is available.

The umpire gave Cowdrey out. But would that have been the decision of a third umpire with digital technology to fall back on?

Specify the mathematical problem

Use information from the photo to argue whether the 'out' decision was correct.

Formulate the mathematical model

Assumptions

Firstly, we need assumptions that will enable us to introduce mathematics into the problem, based on variables we identify as important.

We assume that the batsman achieves a speed (V) at the end of the run that is consistent with a batsman running a sharp single wearing batting gear.

We also assume (supported by the photo) that the bat has been grounded properly.

Most important is an assumption about the path of the bails. Did they fly horizontally on impact, or was there upward movement before descent? The most favourable case for the batsman is the former situation. (Students might discuss why.) This is the situation we will analyse first.

Calculating time

The bail can be seen in the photograph as the dark mark roughly a quarter of the way down the left stump. An internet search gives the standard height of a cricket stump as 28 inches (71.1 cm). The vertical distance, 'h', that the bail has fallen can be estimated by direct measurement of the photograph.

The time (t) taken to reach this point after the stumps were broken can be estimated from the formula $h = (1/2)gt^2$.

(This is an application of the equation of motion

$h = ut + (1/2)\,at^2$

to the vertical movement of the bail, where a, the acceleration is $g = 9.8\ m/s^2$, and $u = 0$ from the assumption of no upward movement of the bail.)

Calculating velocity

Students will need to decide on a realistic estimate of the batsman's speed. How fast can a batsman with pads and bat run over a short distance? An internet search will give students a range of estimates to discuss. For example, finely tuned athletes run 100 metres at about 10 metres per second and 1500 metres at about 7 metres per second. Students will need to make assumptions about the effect of cricket pads on speed, the differences in the type of running between cricket and athletic sprinting and so on (a cricket pitch is only around 20 metres long, sprinters take some distance to accelerate from a standing start to full speed, etc.).

Calculating distance

To estimate the distance that the bat is inside the batting crease as shown in the photo, we note that the distance on the photo between the two creases, the white line of the popping (batting) crease (in front of Grout) and the white line of the bowling crease (that passes through the base of the stumps) must be scaled to represent the actual value of 4 feet (as per the standard dimensions of a cricket pitch). A parallelogram can be drawn, using the shadow of the bat as a guide, that enables the distance of the bat inside the crease to be calculated as a fraction of this distance.

We can then estimate the distance (d) travelled by the batsman in this time at speed (V) using the formula d = Vt.

Now the distance 'd' travelled by Cowdrey since the wicket was broken can be compared with the distance 'r' his bat is inside the crease. This decides the outcome. The umpire raised his finger – do you agree?

Solve the mathematics

Example data

It is best whenever possible for students to identify needed data and source data themselves as the modelling proceeds. They may find relevant sources alternative to those below.

We have drawn on references:
Laws of Cricket: https://www.lords.org/mcc/laws-of-cricket/
World Athletic Records: http://inglog.com/tools/world-records

For the purpose of this resource, an example data set is given below. Students conducting this exercise will likely find different data, and should experiment with the effect a range of values has on their model.

Time since wicket was broken

$h = (1/2)gt^2$

therefore $t = (2h/g)^{\frac{1}{5}}$

$t = (0.3556/9.8)^{\frac{1}{2}}$

t = 0.19 seconds

Distance travelled by batsman in this time

s = Vt
s = 7 × 0.19 = 1.33 m

Position of batsman when wicket fell

r – s
1.10 – 1.33 = –0.23 m

Interpret the solution

The batsman was out by about 23 centimetres.

Consider how outcomes are affected by changes in the values in the model.

Evaluate the model

This can be extended. What if the bails went up in the air after the wickets were broken? Would that be better for the batsman or worse? If the bails went upward on impact the time for the bail to reach its position will be even longer than in this calculation. The batsman will be even more 'out', which also means that there is no need to explore this assumption further for purposes of deciding the question.

What if the batsman was running faster than the speed that was assumed? Our estimation of running speed introduced numerical data not provided in the photo. It can be reasonably argued (again suited to discussion) that the value chosen is likely to be an underestimate. Again, this confirms the decision as the faster the batsman is running the more trouble he is in.

h	distance the bail has moved	7 inches = 0.1778 m	estimated from photo
t	time for which the bail has been moving	$h = (1/2)gt^2$	scientific formula
u	initial velocity of the bail	0	ball was not moving
a	acceleration of bail	$9.8 m/s^2$	gravity
v	velocity of batsman	7 m/s	conservative value based on speed of professional athlete and accounting for effect of cricket padding
s	distance batsman has travelled since the wicket fell	s = Vt	scientific formula
r	distance of the bat inside crease at time the photo was taken	90% of crease width = 0.9 × 4ft 0.9 × 1.2192 m ≈ 1.10 m	estimated from photo
	position of batsman when wicket fell, in relation to crease	r – s	solve for answer to problem

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Further testing of the robustness of the solution can be conducted by allowing the values of measurements made from the photo to vary (say by 10%). Does this make any difference to the outcome?

Note that variations in aspects of the approach are possible and appropriate. For example, rather than estimating velocity up front, calculate the value of V that would lead to the batsman covering the distance of 43 inches in 0.19 sec. Verify it is 5.75 m/s. The argument is then that this is far slower than any Test batsman would run under the circumstances – so he would be out of his ground when the bail was disturbed.

Finally, as a matter of empirical interest, this problem has been used with groups of students at both secondary and tertiary level. The overwhelming response in every case is that the umpire got it right.

Report the solution

The modelling report could contain all the above components of the modelling problem and its solution. The report should synthesise this data into a cohesive narrative, outlining decisions, assumptions and conclusions.