

## Example problem

Level: Junior secondary

Self-generated modelling

# Evacuating



Photo Credit: Jens Sohnrey

Describe the real-world problem

### Australia's tallest apartment block fails fire safety compliance

15 November, 2012. A recent evaluation has identified a potential fire safety hazard at Australia's tallest residential building. The Q1 complex, an 80-storey, \$250-million skyscraper on Queensland's Gold Coast, completed a decade ago, houses more than 500 apartments and more than 1000 residents. The privately-conducted safety evaluation, reported by the ABC, indicated that one of Q1's two emergency escape routes does not meet fire safety compliance standards set by the Australian building code. In the event of a major fire, a design flaw could cause the northern stairwell to fill with smoke, putting hundreds of people at risk of asphyxiation.

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Specify the mathematical problem

A bomb threat is received at 2 am and the building must be evacuated. How long would this take if one stairwell cannot be used?

Formulate the mathematical model

#### Data

This is information that defines important aspects of the structure, and can be provided as background or left for students to source (however websites can give conflicting information). This problem is based around estimates rather than exact calculations and the information below is sufficiently accurate to supports this. The Q1 building contains:

- More than 1000 residents (plus staff)
- 76 residential floors

- 527 residential apartments, including a mixture of 1-, 2- and 3-bedroom apartments
- 1331 stairs per stairwell
- 11 lifts
- 2 stairwells

#### Situational assumptions

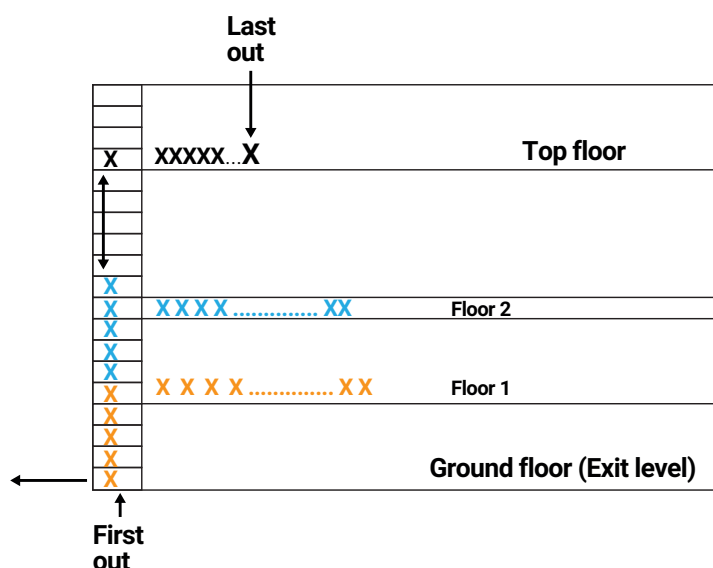
- While building has two exit stairwells, only one is safe.
- Lifts are closed to avoid failure through overcrowding.
- All residents are mobile and hear the evacuation call.
- Building is highly populated and floors have equal numbers of residents.
- 1-, 2- and 3-bedroom apartments are equally distributed.
- Number of stairs between levels is the same throughout.

## Mathematical assumptions

- Number of floors = 76
- Number of apartments per floor =  $527/76 \approx 7$
- Average number of evacuees per apartment (e):  $e = 2.5^*$   
(\*Include 0.5 to represent staff presence throughout building)
- Number of evacuees per floor using stairwell (N):  $N = 7e$
- Number of steps per floor/one stairwell (s):  $s = 665/76 = 8.75 \approx 9$
- Average speed on steps (moving carefully) (v):  $v = 0.5$  steps/sec
- Time delay between successive evacuees when moving (d):  $d = 1$  sec
- Time for first occupant on floor 1 to reach stairs (t):  $t = 10$  sec
- Values of v and d are chosen to reflect a balance between speed and safety.
- Evacuation rate is governed by access to stairs. What happens in corridors is effectively irrelevant because of wait times.

The numbers assigned to the quantities like (e, v, d) need to be estimated, and others (such as 's') inferred. They need to be 'reasonable' in terms of the real context, and can be varied for different calculations.

For example, the average number of evacuees per apartment has been chosen with a fairly 'full' building in mind. Average speed on steps, and time delay between people moving, need to take into account that people crashing into one another must be avoided, while otherwise moving as quickly as possible. Values here can be agreed by discussion, and/or acting out. (Students have used thought experiments effectively here.) Similarly, selection of other numerical values needs to be supported by discussion.



Total time (T) for evacuation = time for first occupant on floor 1 to leave building + delay until last occupant can move + time for last occupant to leave building (this needs thinking about).

Show that:

$$T = (t + s/v) + (fN - 1)d + fs/v \text{ where } N = 7e$$

## Solve the mathematics

$e = 2.5$ ;  $f = 76$ ;  $s = 9$ ;  $v = 0.5$ ;  $d = 1$ ;  $t = 10$  gives  $T \approx 45.4$  min

For a lightly populated building assume  $e = 1.5$ ; for faster or slower steps vary 'v'; for different separation distances between moving evacuees vary 'd' and so on.

## Interpret the solution

Consider the time taken to evacuate residents from the building under the various assumptions. Describe the influence of the numerical changes in assumed values? What are the risks? What kind of robust estimate seems reasonable? Is it likely that the residents could exit safely in the event of a fire?

Estimate how much more quickly the building could be evacuated if both stairwells were safe and operational. Is it likely that residents could exit safely if both stairwells were functional?

## Evaluate the model

Consider how outcomes are affected by changes in numbers of people, speed of descent and delay times. Are these what you would expect? What does the model suggest are the most important influences on evacuation time? Is this consistent with intuition and experience?

## Report the solution

A simple modelling report will address the time question as it is written in the problem statement. It should contain all the above components of the modelling problem and its solution, including implications of further calculations using a range of different parameter choices. A more comprehensive report, going further than the question demands, would synthesise these data into a cohesive narrative, considering the implications for safety of the residents of the apartment building, and considering if possible, fire safety compliance standards set by the Australian building code. The precise structure of a report will depend on how the problem was expressed.