2 international mathematical modeling challenge

Information Booklet

www.immchallenge.org.au



About the International Mathematical Modeling Challenge (IM²C)

The IM²C is a team-based mathematical modelling competition designed to promote mathematical modelling for students at secondary school level.

Held annually on a global scale, the IM²C has a national component for students in each participating country, and an international component where the best work from each participating country is assessed by an international judging panel, with prizes awarded at both levels.

Operating in teams, comprising up to four students from the same school, participants must work collaboratively to solve a centrally-set problem by devising and applying an original mathematical model.

The competition takes place during a fixed period each year, usually between March and May. During the competition period, teams nominate a period of five consecutive days in which to develop their solution to the problem. Under the supervision of a team advisor (usually a teacher) teams unpack the given problem, hypothesise, test and develop a working solution to the problem, before preparing a report that is <u>submitted for judging</u>.

Judging occurs firstly at a national level, with the best reports from each

participating country progressing to be judged internationally.

Aim

The main aim of the IM²C is to promote mathematical modelling, encouraging participants to explore the application of mathematics in real situations to solve problems of importance.

Encouraging an extension of experience in mathematical modelling for students in secondary schools, the IM²C seeks to develop and enhance students' ability to visualise, understand and apply mathematics in real-world contexts, providing a valuable opportunity for the practical demonstration of learning and application of theory. By mobilising students in teams, the IM²C replicates real-world conditions; requiring collaboration and contribution from different skill sets, perspectives and methodologies to achieve overall success. Providing an opportunity for peer-based learning, the IM²C addresses the four sub-strands of the Mathematics Proficiency Strand outlined in the Australian National Curriculum -Understanding, Fluency, Problem-Solving and Reasoning - as students work together, communicate with one another and employ creativity, reasoning and logic to successfully solve the defined problem.



For official competition dates and more information, including examples of modelling problems and solutions, and other support material, please visit **www.immchallenge.org.au**

What is mathematical modelling?

Mathematical modelling as real-world problem solving

Mathematical modelling is:

- a process in which real-life situations and relations in these situations are expressed by using mathematics ¹;
- a cyclical process in which real-life problems are translated into mathematical language, solved within a symbolic system, and the solutions tested back within the real-life system².

In both instances, mathematical models are seen to move beyond the physical characteristics of a real-life situation to examine its structural features through mathematics. Mathematical modelling entails the construction of mathematical models of natural and social phenomena that are problem-driven, where the choice of relevant mathematics is itself part of the problem-solving process.

Mathematical modelling: Concept and theory

From a theoretical point of view, the concept of mathematical modelling can be represented in three parts (Fig.1):

- i. an extra-mathematical domain (D)
- ii. some mathematical domain (M)
- iii. mapping from the extra-mathematical domain (D) to the mathematical domain (M), resulting in outcomes that are translated back to the extra-mathematical domain (D).

The 'extra-mathematical' domain (D) is a useful way of representing some part of the 'real world' that is relevant to a particular problem.

Objects, relations, phenomena, assumptions, questions and the like in D are identified and selected as relevant for the purpose or situation and are then 'mapped' – translated – into relations, phenomena, assumptions, questions and so on pertaining to M.



Figure 1: The iterative mathematical modelling cycle

Within M, mathematical deliberations, manipulations and inferences are made, the outcomes of which are then translated back to D and interpreted as conclusions concerning the problem in the context of that domain.

These activities are often aided by technology that enhances the mathematical process, performing tasks or calculations in cyclic fashion. This so-called modelling cycle may be iterated several times on the basis of validation and evaluation of the model in relation to the domain, until the resulting conclusions are satisfactory in relation to the purpose of the model construction.

¹Verschaffel, L., Greer, B., & De Corte, E. (2002). Everyday knowledge and mathematical modeling of school word problems. In K.P. Gravemeijer, R. Lehrer, H.J. van Oers & L. Verschaffel (Eds.), Symbolizing, modeling and tool use in mathematics education (pp. 171-195). Dordrecht, The Netherlands: Kluwer Academic Publishers. ² Haines, C. & Crouch, R. (2007). Mathematical modeling and applications: Ability and competence frameworks. In W. Blum, P.L. Galbraith, H. Henn, & M. Niss (Eds.), Modelling and applications in mathematics education: The 14th ICMI study (pp. 417-424). New York, NY: Springer.

A framework for mathematical modelling

The IM^2C operates on the assumption that opportunities to use mathematics are everywhere in the world around us. The challenge is to identify those opportunities, access them and apply mathematics productively to better understand the situation that confronts us and to resolve the problems presented. The IM^2C exists to help students:

- develop a systematic and successful approach to addressing individual problems located in real-world settings, and;
- through this development, enable students cumulatively to become effective solvers of real-world problems.

The aim is to develop students who not only can address problems set by others productively, but also become able to identify and address problems themselves.

In order to be useful and applicable in practice, both in the context of the IM²C, and more broadly, the cyclic process of modelling is scaffolded or guided by a systematic approach to individual problems, consistent with the approach taken by professional modellers when devising solutions to problems in their field. This systematic approach involves seven stages.

- 1. Identify a (real-world) problem
- 2. Specify a related mathematical question(s)
- Formulate a mathematical model to address the question (including making simplifying assumptions, choosing variables, estimating magnitudes of inputs and so on)

- 4. Solve the mathematics
- 5. Interpret the mathematical results in terms of their realworld meanings
- 6. Make a judgement as to the adequacy of the solution to the original question(s)
- 7. Report on success or make adjustments and try for a better solution

Stages 6 and Stage 7 serve to indicate the cyclical nature of mathematical modelling.

If the proposed first solution is not an adequate solution to the original question, the problem needs to be readdressed by repeating earlier stages (3–6) in sequence. This may need to be carried out several times before an adequate solution is found.

Sometimes an extension or refinement of the original problem is suggested by the outcome of a first modelling endeavour. In this instance the question is re-specified, and further cycles of activity are conducted with the new question.

It is also important to note that although the stages are sequential, the cycle is not necessarily smooth, as the constant checking, testing and evaluating contained in each stage means that there is frequent movement within, and between, the stages – potentially making the development of some models a very challenging exercise.



To illustrate the processes typically employed in mathematical modelling, consider the task of adapting a recipe for a greater number of people than originally intended.

The problem is simple, but it serves to illustrate the steps and processes that are required for problems of far greater complexity.

Chocolate Mousse 4-6 people.

185g of cooking chocolate ¼ cup of hot water 5 large eggs 1½ cups of cream

1 teaspoon of vanilla essence

Problem: Adapt the recipe to cater for 6-9 people

Most of us would adapt this recipe using the back of an envelope or even in our heads with possibly a calculator to help us. In doing so we would probably overlook that we have made assumptions that are so 'obvious' that we did not realise that we had made them.

Using a common spatial representation of the modelling process – a seven box diagram (Fig. 2) – we can visually represent an approach to the problem, with arrows between boxes indicating the movement through the modelling cycle:

Figure 2: The seven parts of the mathematical modelling process



7. Communicate/Report

Use the model to explain, predict, decide, design, recommend and so on. The amended recipe written out with assumptions explaining how it was approached. Figure 2 illustrates how key elements of the solution process for this problem can be represented and linked to a general approach to mathematical modelling.

This can be useful for structuring a report. In the formulation box we note essential assumptions that underpin the model development. Identifying them explicitly in such a simple case helps to emphasise their centrality to every modelling enterprise. Similarly in the interpretation box real world practicalities moderate precise mathematical results, a step that can easily be taken without recognising it. The three arrows pointing out of box 6 reflect that (in the general case) evaluation may be followed by a report, but may require instead a revisiting of the problem context, the mathematical problem identified – and indeed other stages of the modelling process.

Additional simple and bridging modelling problems

To help bridge from simple modelling problems to more complex problems, the IM²C website contains a number of examples for download and use in the classroom. These problems range from the senior primary/junior secondary level through to senior secondary, and finally to examples of IM²C level problems such as the one outlined overleaf. For additional mathematical modelling problems and challenges, including example solutions, please visit www.immchallenge.org.au/supporting-resources

IN²C Problems

The problems used for previous years are shown on the IM²C website: https://www.immchallenge.org.au

As selection of team solutions is also shown, and commentary from the judging panel (both the Australian and International panels).



Preparing a team

When preparing a team to enter the IM²C, it is important to ensure that all students have access to computers that enable them to word process, use a spreadsheet, and dynamically model both graphs and geometric relationships. Students should also have access to graphing calculators.

All parents of students participating in the contest must sign a Parental/Guardian Permission Form making their child's work available (with confidentiality assurance) for use in example and training materials for IM²C professional development activities. Additionally, should their team be designated as an Outstanding winner, the team's Solution Paper (or solution abstract) will be published on line.

When registering for the IM²C, be sure to use a valid current e-mail address so that we can use it to contact your team at any point before, during, or after the contest, if necessary.

Each team will be assigned a control number/identifier as part of the online registration process. This is unique to your team, and identifies you as participants in both the Australian Round, and subsequent rounds of the International Mathematical Modeling Challenge.

Team advisors are encouraged to coach and prepare teams in advance for the $\rm IM^2C$, with sample problems and supporting materials being made available as the commencement date draws closer.



during the operation of Australian Daylight Saving Time (which is UTC/ GMT+11 in the Eastern States), and concludes during the period of the year that uses Australian Eastern Standard Time (UTC/GMT+10 in Australia's Eastern States). Care should be taken to note the local time relevant to events of the Challenge.

Rules

- A team may consist of up to four secondary level students, enrolled in the same school.
- The contest runs during a specified period each year. The dates and times for the contest period each year are announced on the IM²C website.
- Each team will decide on a consecutive 5 day period during the contest period to work on the contest problem. For example, the contest for your team could be from 8:00 a.m. on a Thursday after the beginning of the contest period until 8:00 a.m. on the following Tuesday. Your team's contest ends upon completion of the 5 day period.
- You must obtain and use your control number and password to participate in the IM²C. Team advisors log on to the IM²C Australia web site to register the team's intention to participate. Team registrations open early in each school year.
- Then, at the beginning of their 5 day contest period team advisors log on to the website to receive the contest problem.
- Team advisors will be given access to the modelling problem at that time, or as soon as they log in after that time.
- Team advisors must ensure that no alterations of any kind are made after the end of the team's 5 day period. Team reports must be collected from students at or before this time and uploaded to the IM²C website as shortly after.
- All team reports must be received at the Australian judging centre no later than 5:00 p.m. on the final day of the contest period for that year.
- A signed Parental/Guardian Permission form must be included for each participating student. Schools should send the original signed forms to the Australian Council for Educational Research at the address below, and retain a copy of each for their records.

International Mathematical Modeling Challenge c/o Australian Council for Educational Research Private Bag 55 Camberwell VIC 3124

Complete rules, and detailed sumission instructions can be found on the IM²C website by visiting www.immchallenge.org.au/registration-and-rules.

Getting involved in the IM²C

Completely free to enter, the IM²C exists to support the realworld application of learning, build proficiency, encourage collaboration, and challenge students to use mathematics to make a real difference to the world around them.

All Australian schools are invited to enter. Enquiries about the IM²C in Australia can be directed to **contact@immchallenge.org.au**

For more information and to register for the IM²C please visit: **WWW.IMMChallenge.org.au/register**



The International Mathematical Modeling Challenge (IM²C) in Australia is organised and implemented by the Australian Council *for* Educational Research (ACER), under the guidance of a national advisory group.

