Australian Judges' Commentary for 2021

The Australian IM^{2}C judges congratulate the 74 teams from 31 schools all across Australia who participated in the 2021 Mathematical Modelling Challenge. One of the two highest ranking Australian teams gained a Meritorious Achievement in the International Mathematical Modelling Challenge in which 51 teams from 27 different countries participated.

The 2021 Challenge

The challenge this year was to determine who is/was the greatest athlete of all time (G.O.A.T.) in a particular sport. There were three parts to this multi-faceted challenge.

1. Teams had to develop a mathematical model to choose the best female tennis player of 2018, given data of the last four rounds of each of the four grand slam tournaments.

2. Next, they had to choose another individual sport or sports division, find relevant data and develop a model to choose the greatest athlete of all time in that sport.

3. Finally, they also had to explain how this model from part 2 could be adapted to an individual sport of any kind and to team sports.

Each team had to describe their methods and results in a main report of up to twenty pages, a one-page technical summary and a one-page non-technical letter to a sports director. Any supplementary information, data and code could be added in an optional appendix which was not included in the judging process.

This was a very challenging task because there were many factors to consider and difficult decisions to be made. Data collection, modelling and report writing are all time consuming tasks, even if all tasks go to plan. Here are some observations and advice from the Australian judges.

Problem Interpretation

Most teams had reasonable interpretations of the problem and distinguished between the performance during a one-year season and performance over a multi-year career. Some teams discussed factors such as popularity or social impact that could not easily be measured or modelled in a non-subjective way. Some teams thought greatness could be achieved in a single, record-breaking event, but most thought that persistence and consistency of high achievement were also necessary.

Solving the Initial Problem

There were several approaches to the first part of the challenge: women's singles tennis grand slams. Although most teams used only the supplied data, a few augmented this with additional data and statistics from official websites. Most teams developed descriptive models of tournament results as various weighted combinations of rankings, matches, games, sets and points won and lost. Model parameters were mostly chosen to reflect the different levels of winning in different rounds of a tournament, but sometimes the choices seemed arbitrary and not backed by evidence. Some teams tried to account for missing data from particular athletes by attributing the missing data to losses due to the knock-out tournament structure, injury and non-participation.

A few teams tried a parameter-free approach by modelling the results of tournaments as weighted or unweighted directed networks, with varying success.

Unfortunately, a few of the more sophisticated models contained serious numerical instabilities that went unnoticed and unchecked by the team. Some teams used relatively advanced mathematical methods but failed to understand the underlying principles or applied them inappropriately. Judges would like to remind participating teams to check their models for instabilities and ensure they fully understand the techniques they are applying.

One team applied two known modelling techniques of ranking competitors in tournaments, one adaptive and one non-adaptive, and compared them to simpler, more common techniques to assess their value and validity. Though these techniques were not original, the application of these modelling techniques to the Challenge problem did contain original elements. The application of the various modelling techniques was explained reasonably well.

There were no entries that explicitly used the logic of probability, information or model complexity to evaluate and compare different models.

Solving the Main problem

The second, main part of the challenge, was to develop a model to determine the G.O.A.T. in a self-chosen individual sport. Surprisingly, the easy choice of men's singles tennis was rarely made. A wide variety of individual sports were chosen; some based on sequential comparisons between players and others on measurements of physical quantities.

Apart from all the previous considerations, this part of the challenge also involved modelling performance over time. Definitions of greatness included: average over time, improvement over time, highest level attained, distance from theoretic perfection, and greatest cumulative achievement over time. Some teams defined their sports by the availability of data and this was acceptable when adequately justified. Some teams considered the significance of major rule changes on the definitions of their sport. One team modelled the effects of weather conditions on the outcomes of road races to compare athletes participating in different events.

The time and effort involved in finding and managing real-world data may have been underestimated by many teams. Factors such as match fixing were not considered by any of the teams. Some teams included their entire data sets in an appendix or supplied references to sources, but many did not adequately indicate where the data was from, nor how it was processed.

Extending the Model

The final section discussing adaptations of the model to other individual sports and to team sport, were generally less well done. The models of the previous sections were sometimes acknowledged to be too closely linked to the idiosyncrasies of particular sports for them to be easily adapted to others. Some teams did manage to suggest reasonable adaptations to sports of a similar kind or explained why it would not be possible to adapt the model to other sports, but discussions were usually too brief and suggested ideas were vague or impractical. The judges' felt that teams likely ran out of time when they reached the final section of the Challenge.

There were some interesting ideas about adapting the model to team sports. Some teams interpreted the task to be determining the greatest team of all time. Others considered the specialised roles within teams, cooperation among players and how to allocate credit, etc. One team wondered whether the two-car Formula One racing teams or the individual drivers in each team should be compared, since different teams have significantly different budgets and equipment. Again, discussions mostly tended to be brief and vague.

General Advice

Judging follows a comprehensive list of general criteria including problem interpretation, model formulation and explanation, interpretation and evaluation of results, discussion of model limitations and adaptations, and overall quality of the reports. The mathematical aspects of the challenge are most important but the overall quality of the writing and logical structure of the report also create a powerful impression.

Based on the submissions of the current and previous challenges, we wish to make the following specific recommendations:

1. First spend some time defining the exact requirements of the task and ensure that the entire team understands what each member needs to do. Small misinterpretations at the beginning lead to confusion in the end, and can result in time consuming fixes.
2. Allow more than enough time for writing; it takes longer than most people think. Sketch an outline with headings early and create a writing schedule to stay on track.
3. Brainstorm and strategize together. Concentrate on big ideas first. Be clear about what kinds of problems need to be solved and what steps need to be taken. Use sketches, diagrams and short verbal descriptions but avoid mathematical language until there is a clear understanding of what to do.
4. Specify the problems, assumptions and constraints as precisely as possible. Models are approximations of reality, so decide what must be included and what can be ignored. Leave out all that cannot be measured. Explain these decisions clearly in the report.
5. Start developing appropriate mathematical models based on available data only. Start with simple and understandable models. Study what each part does and how the parts combine. Compare different approaches. Use small sets of data to test how the models work and behave so that small changes may be made earlier rather than later. Time is very important.

6. Rather than using fixed, arbitrary parameter values, even if they seem reasonable, try to use parameter values that are determined by the data. For example, try to find models that are functions of known data and can predict values of yet unknown data as well as possible. In the tennis problem, for example, what combination of data from earlier matches in a tournament predicts the winner of a following match most often? Such a model justifies itself.
7. Do not use unnecessarily complex methods to impress the judges. Only attempt more sophisticated methods or models if they can be explained clearly and concisely, and their use is justified by the context. Be very careful about this. Remember that complex models tend to be better than simple models at representing known data but worse at representing new data, which is usually what is required.
8. When satisfied with the models and methods, decide how to communicate with the readers. Introduce ideas in a logical order. Introduce all variables clearly. Choose variable names carefully so that they remind a reader of their meaning. Do subscripts of sequence variables make sense? Do they start from zero or from one? Be consistent. Don't switch notation between sections of the report. Number equations if they are referred to in the text.
9. Explain how methods work using simple language, simple diagrams or simple examples so that the readers can follow with the least amount of effort. Ask yourself, using your descriptions, would another person be able to implement the methods and get the same results?
10. Interpret the results. Do they make sense? Have they answered the questions? What do they mean? How do the methods and models compare to others? Can they or should they be changed or improved? If so, how or why? Think carefully.
11. Check and edit the final report as a whole. Use the same formatting for all sections. Avoid repeating the same ideas in different sections. List the relevant and accessible source material.
12. Place only non-essential material such as data tables and computer programming code in the appendix. Judges may sometimes refer to this material but do not consider it in the judging process.

In summary, we congratulate all participating teams on their efforts during this challenge, we thank each school for their support and we hope every team member will continue to improve their skills in thinking and communicating mathematically. The originality of the models developed was inspiring, and we look forward to seeing the entries to the 2022 Challenge.