2024 International Mathematics Modelling Competition (IM^{2}C)

Summary Sheet

In this paper, our team makes a quantitative scoring system that decided the readiness of any given household to own a pet, as well as determining the number of future pet-owning households globally. Such a problem was brought to our attention by the IMMC-A due to the unpreparedness of many families in buying a new pet for company, which spiked during the COVID-19 pandemic, causing a huge detriment to both households and animals alike.

Firstly, **radar charts** easily allowed a range of factors, which were standardised to percentiles using **distribution functions**, to be considered in creating an ideal condition for animals in a household. Thusly, comparison of a household's conditions to the ideal could be achieved through comparison of two polygons. The **Shoelace Formula** was applied to assess the similarity of these polygons through calculating magnitude of overlap, as well as rewarding households for having an even more suitable environment for a pet, at a diminishing rate. It was determined that a household was ready to own a pet when a similarity score of 75%. Furthermore, these factors were used to calculate the number of households in any given region to own a pet.

The model was then applied to the suitability of six unique households desiring to own a cat, using data obtained from repeated research. Three households passed, and the other three failed, due to shortcomings in multiple factors. Due to the generalisability of the model, the suitability of owning a dog, goldfish, horse, and parrot were also determined for these same households. Further, it was found that the current number of cat-ready houses in **New York was 2,463,105, 453,330 in Singapore, and 35,208 in Copenhagen** by generalising these methods to a wider population.

One factor that had not been considered was the compatibility of the pet being desired with other possible pets already in the household. Therefore, an **Inter-Species Compatibility** score (ISC score) was inspired by **dominance matrices** methods, finding the compatibility using attribute dominance and showed that as the number of pets in a household increase, the less ready a household is to take in another one.

Considering all these possible factors, as well as the willingness of households to own a pet, the future of pet-ownership households in New York, Singapore, and Copenhagen was modelled using **differential equations**. Differential equations, in conjunction with a **General Readiness Function**, allowed for the rate of change for different factors to be considered. Overall, a general pattern was established in each area. The number of pet-less households decreased quickly, corresponding to a sharp rise in households with one pet. The rate of change of both slowed as the number of households available to get a pet decrease, and the proportion begins to fit the "natural" proportion dictated by parameters. The number of households with two pets steadily raised, though the number of households with three or more pets usually decreased, given that the loss rate tends to outpace the percentage of the population able and willing to acquire a third, fourth, or even fifth pet.

Finally, the adjustability of the model was assessed through a **sensitivity analysis.** We concluded that our model would be relatively insusceptible to inaccuracies in input, and as such is robust in handling potential common changes to household situations.

Letter to the Decision Makers

Dear IMMC-A,

Our team has arrived at a final model that allows for pet shops and animal shelters to calculate the readiness of a household to own a pet. We think this model is thorough and effective in arriving at a definitive conclusion, due to the consideration of many major factors relating to pet-household and pet-pet compatibility.

Using quantitative data of the most crucial factors – materialistic, non-materialistic and location - for the health of domesticated animals and pet owners, we developed a comprehensive model as explained in our summary sheet; it maximised comfort and satisfaction and minimised abandonment. Our model used geometric methods to determine compatibility, which you can easily analyse visually and iterate many times through automation, as shown in our examples. We also investigated the effect of multiple pets and species in one household and recommend the ISC model to be used when encountering this scenario to accurately evaluate readiness.

We recommend a passing benchmark of 75% readiness in our assignment for pet-household compatibility and 50% in pet-pet compatibility; however, these can be easily adjustable for your requirements. The effects of changing this benchmark have been explored in the report. Also, for your convenience, the model produced is easily adjustable to any pet and household though we have left an example about cats based on your request. Additionally, if you wanted to use regional data and percentiles, this can be done by simply changing the mean of each distribution curve we listed.

You further asked for the model to calculate the number of houses in any given region that owned a pet. This required only minor generalisations from the specific case of the model and applied the passing benchmark to all household metrics. It was suggested that only the five strongest factors would be considered in this process as not all households are completely well-rounded.

Lastly, our model allows for the number of houses owning any pet in any region to be determined and predicted through observing the rate of change of different household factors. This includes household willingness to own a pet, as well as percentage of the population meeting the benchmark for at least 5 factors, the population and household number growth, biological death rate, and carrying capacity of each region. This report predicted future pet demographics across . Again, you can easily adapt this model to different regions and pets by simply changing the basis of all our models – the factor parameters.

We reassure you that minor errors in household inputs will not majorly change the outcomes of a household's readiness score, as a sensitivity analysis using mass simulation of data input was conducted to ensure there was a low number of erroneous successes, even with 30% input error.

This recapitulates the main aspects and conclusions from our finalised model. Thank you for trusting us with this task, and we hope you now have more insight into this tedious problem.

Kind regards,
Team Radar

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1.0 Introduction

Pets provide care and social support for humans; it is a well-known fact. You only must remember the loneliness and isolation during the COVID-19 pandemic to prove this. During those challenging times, the number of households with pets rose substantially, further stressing how pets offer companionship and comfort. However, new, or unprepared households were not ready for the complications they would face in a pet-human relationship, leading to increased health risks for both humans and their animals, pets being returned to pet shops and shelters, or even abandoned. Therefore, we have decided on a quantifiable measure to determine whether a household is prepared to own a pet designed for easy use by the International Mission for the Maintenance and Care of Animals (IMMC-A).

In this report, a model is developed to determine the suitability of a household to own any specific pet. It was decided that a radar chart easily and clearly allowed for situational comparison. Factors required for a quantifiable analysis were researched from reliable sources, with factor scales standardised based on statistical analysis. By conducting further research, the model was applied to six different households desiring a cat, with results showing that a better suitability was achieved through either moderate strength in each factor, or greater strength compensating for weaker areas. However, it was easily observed that the model output greater readiness scores for the former situation, indicating the model's preferences.

With a model created, it was required for the model to be applied to different regions or countries, to determine the number of households suitable for a cat in that region, thus proving the generalisability of the model to both different households. To further demonstrate the ability of the model to be generalised for a variety of pets, the model was applied to four other animals of different factor strengths, and the six different households. This introduced another factor, the compatibility of a pet with another pet, which was calculated using dominance matrices methods. Lastly, to test the reliability of the model, future trends were observed through use of differential equations. A sensitivity analysis performed evaluated determined the model's adjustability.

1.1 Definitions

- A pet is a domesticated non-human, non-plant biological species kept by a household for non-utility reasons, likely for companionship.
- Used interchangeably with preparedness and suitability, readiness is the quality of being willingly able to perform a task. In this paper, the extent to which a household is ready to own a pet was decided with a percentage score, S.

1.2 Mathematisation of the Task

This task required us to create a suitability model that gave a quantity that decided whether a household was prepared to own a pet. To achieve this, a combination of multiple factors was standardised (to be expressed as percentiles) and formed into a graph allowing easy and intuitive visual comparison. This comparison could then be stated mathematically as a score. An ISC (Interspecies Compatibility Factor) was then developed to quantify the ability of multiple pets to co-exist within a household.

1.3 Variables and Parameters

Notations	Descriptions
F	Daily Free Time (hours)
H	Household Space (square metres)
I_d	Annual Household Disposable Income ($/year)
A_c	Total Household Animal Care Experience (hours)
T	Temperature Difference (°C)
D_v	Distance to Vet from Household (km)
A_{si}	Area of the "ideal shape" of a pet
A_{sh}	Area of the "shape" obtained for any individual household
A_{so}	Area of the overlap between A_{si} and A_{sh}
A_{se}	Area of the region of a household's shape outside of the "ideal shape"
A_{ea}	The bonus to score derived from excess area
S_{ea}	Excess Area Score, as a percentage
S_h	Pet-Household Compatibility Score, as a percentage
S	Measurement of similarity between two metrics
S_p	Pet-Pet Compatibility Score, as a percentage
P	Probability of a given house passing S_h
$D_{\text{x Score}}$	Pre- and post-multiplication by row and column matrices of 1s for $\boldsymbol{D_x}$
$D_{\text{Max score}}$	Pre- and post-multiplication by row and column matrices of 1s for $\boldsymbol{D_{\max}}$
P_n	Number of households with *n* pets
L	Loss rate of pets per year
$R(n)$	Generalised Readiness (Proportion of the population ready for *n* pets)
$W(n)$	Willingness (Proportion of the population willing to have *n* pets)

1.4 Assumptions and Observations

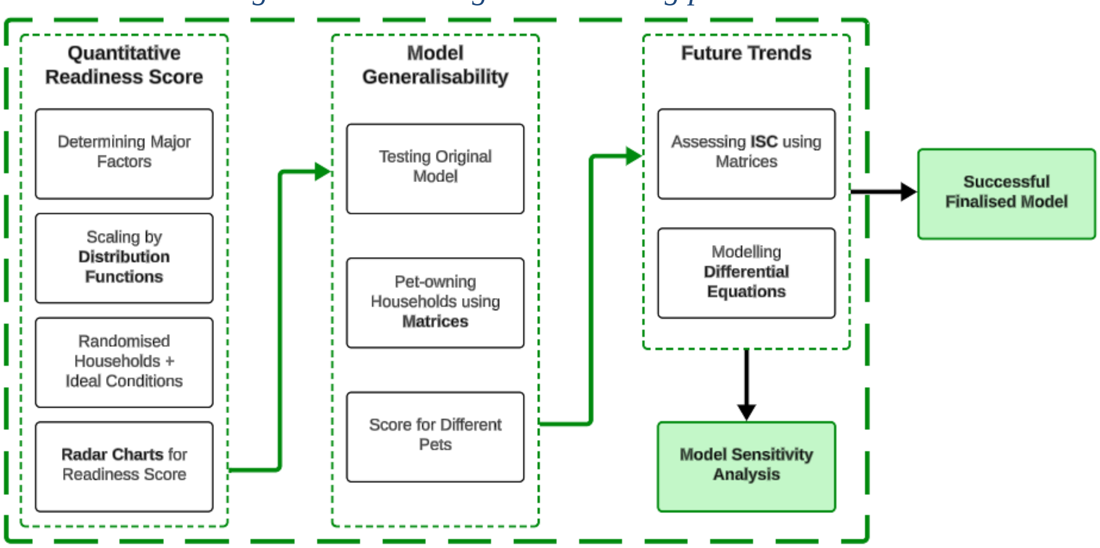
A vital component of the model was its ease of use, and thus assumptions were made to simplify the model while retaining accuracy, as many factors contribute to a household's readiness to care for a pet.

- It was assumed that pet ownership was not shared between households. This would unnecessarily complicate the model, when here readiness can simply be found by analysing each household separately.
- It was assumed that the household was willing to dedicate the time and effort required to take care of the pet (if such time was available, as considered later). Willingness is exceedingly difficult to measure prior to pet ownership, and so is not considered. This introduces minimal error, however, as it is positively correlated with strength in other factors.
- It was assumed that the pet was solely for companionship and enjoyment purposes, as if a pet serves some other function such as assistance, readiness becomes of second importance, while this would also add a vast number of additional factors.
- It was assumed that no members of the household were allergic to the pet, as trivialising factors should not need to be considered in the model – evidently, an allergy should always prohibit readiness.
- It was assumed that all relevant data input accurately reflects their conditions to avoid attempting to unnecessarily account for uncertainty, though as shown later, the model is resilient to uncertainty.
- It was assumed that animals occupy only one environment (e.g. fish tank or in a living room), and that animals living in different environments do not environmentally interfere with each other, so that the ISC does not indicate conflict between two pets which can never interact with each other in real life.
- It was assumed that there are enough pets to provide for every ready household, so that supply (which is clearly not in danger of running out) does not have to be considered.
- It was assumed that, in the future, each household would seek only one kind of pet. This assumption, while unreasonable, was made to create a possible solution, as the dynamics of each pet's popularity based on a household's previous pets are highly unpredictable. Note that this only applies in Part III.

Some observations were then made:

- Qualitative input should not be implemented as part of the model, as it is impossible to convert such data into numerical values while retaining full meaning and nuance, indicating that such factors, such as household stability, should be considered through logical correlations with other factors.
- Animals under the same genus, by default, can co-exist relatively well within the same household, meaning that a successful ISC should indicate high values for households with pets of the same type.
- Future trends for pet ownership depend on a nearly limitless number of factors, and so only the most crucial factors – pet mortality, pet uptake willingness, and pet readiness – will be considered.

Figure 1 - Flowchart illustrating decision-making and modelling process.



2.0 Cats Model

2.1 Determining Major Factors

To obtain accurate and reliable data, many resources consulted online were from reputable sources (e.g. Animal Medicines Australia [1] and Australian Government). Six major quantifiable factors, categorised by materialistic, non-materialistic and locational were chosen to cover the question of pet readiness. There were many factors that also could been included, such as the household demographics and living conditions. However, due to the decrease in user-friendliness and great increase in model complexity, these factors were ignored on the basis that strong scores in these dismissed factors would correlate with strong scores in factors which were included.

- **Free time (non-materialistic):** Denoted by F in minutes. Companion animals thrive on human company and will depend on their owners. Potential owners who have busy lifestyles need to carefully consider their capacity to provide adequate care. Lack of care time would be detrimental to pet health.
- **Household space (materialistic):** Denoted by H in m^2. All animals are prone to sleepiness, irritability, and personal boundary violations. The ability to give pets space should be a key part of pet readiness.
- **Disposable income allocated to pet (materialistic):** Denoted by I_d in \$. Medical costs, whether one-time such as vaccinations or ongoing such as pet food, are a major part of care for a pet. For a household to be ready for a pet, their disposable income must be sufficiently high for that pet's needs.
- **Animal experience (non-materialistic):** Denoted by A_c in months. Looking after a pet requires thorough sufficient research, even before purchasing one. This includes previously owning a pet, or working in animal/service industries, which greatly raises the level of care an owner can likely provide.
- **Temperature Difference (locational):** Denoted by T in °C**,** which represents the difference between average outdoor climate and standard room temperature. Animal behaviour and physical welfare is dramatically affected by temperature. Pet-ready households should thus be located in areas with minimal temperature difference (as practically all household pets are comfortable at room temperature).
- **Convenience of Achieving Health and Satisfaction (locational):** Denoted by D_v in km. It is important for pet owners to be within proximity to health services and pet shops, for convenience, pet welfare and satisfaction. Without this, a sufficient level of care for a pet is not achievable.

2.2 Scaling by Distribution Functions

The primary tool used to determine pet-readiness of a household was a comparison of scores to ideals, using a simple and visually intuitive tool: **the radar chart** [2]. As in the example above, each factor outlined previously would form one "axis," thus developing a different shape for each household based on their scores in each factor. These shapes could then later be compared to demonstrate a level of suitability.

Firstly, it was necessary that a scale was developed for each factor, as the radar charts produced should be clear and easily legible, and thus an intuitive visual comparison would match mathematical results. Additionally, all measures and scales must be standardised to ensure each factor is comparable. This was done simply by collecting percentile data for each factor, and setting the scales from 0 to 100, with the number indicating which percentile a given household sits in compared to the global sample. For example, with a floor area of $20m^2$, this could place the household in the lowest 11% of households. After extensive research, the following cumulative distribution functions were determined for each factor:

Table 1 – Cumulative Distribution Functions for percentiles of each factor

Factor	Type	Score Calculation
F	Normal	$\int_0^F \frac{1}{21.92322572\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{F-63}{21.92322572}\right)^2} dF$
H	Normal	$\int_0^H \frac{1}{22.59850327\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{H-35.79334401}{22.59850327}\right)^2} dH$
I_d	Calculated With Regression	$-1.75918 \times 0.999832^{I_d} + 1.75511$
A_c	Normal	$\int_0^{A_c} \frac{1}{72.64640474\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{A_c-192}{72.64640474}\right)^2} dA_c$
T	Normal, Inverted	$1-\int_0^T \frac{1}{5.32374870372463\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{T-7.150880829}{5.32374870372463}\right)^2} dT$
D_v	Normal, Inverted	$1-\int_0^{D_v} \frac{1}{22.03097488\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{D_v-31.58707651}{22.03097488}\right)^2} dD_v$

These formulas were calculated through basic statistical analysis. First, it was observed that generally, normal distributions served to model the global trends. Secondly, the mean of each factor was found through research, thus enabling the use of the normalisation formula:

$$z = \frac{x-\mu}{\sigma}$$

Percentile data was converted to z-scores (z), and corresponding values, the x-scores, to each percentile were thus used in the formula to calculate standard deviation, which was averaged if multiple percentiles were used in calculation. This allowed the construction of a distribution function for each metric, allowing a consistent scale to be used for the remainder of the paper. While **animal experience**[1][9], and **climate tolerance**[10][11] had readily available data showing an approximately normal distribution, **free time**[3]-[5] and **disposable income**[8] required a conversion to the share of them used on pets, as obviously no household is likely to dedicate the entirety of their free time (including social activities, hobbies, and entertainment) to their cat. Interestingly, **disposable income** did not match a normal distribution, and so regression was used to find a distribution function for researched data with a high correlation coefficient, r^2 of 0.9739.

The **convenience** factor used data of distance from dwellings to hospitals (which presented a similar density to the number of vet clinics) from multiple population centres as direct data for vet clinics specifically was unavailable. Notably, this factor scores well for lower distances, meaning that higher percentiles are associated with lower distances, the opposite case from the other factors. This logic was also applied to **temperature difference**, as a smaller difference to the ideal temperature is better. **Household area** [6][7] percentiles were also unavailable, but as floor area is strongly correlated with net income with an r^2 value (determination coefficient) of 0.9646 (see appendix 2), percentile data for income was translated to floor area, resulting in a valid normal distribution.

2.3 Ideal Cat Conditions

The next step was the construction of an "ideal shape" or graph for the set of scores which represented the ideal household for an animal - in this first section of the paper, a cat. Using the translation formulas from above, the data for ideal cat environments was transformed into scores, thus constructing the "ideal shape" for the cat. This data was readily available for the cat and a number of other animals, indicating that the model was easily generalisable between pets (explored more below).

Figure 2 - Radar chart of ideal cat conditions.

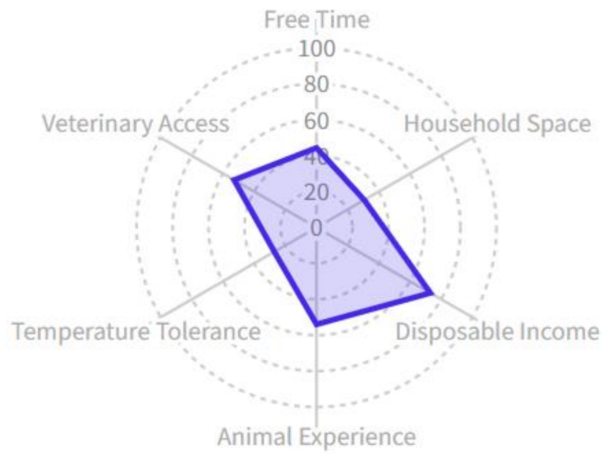


Table 2 - Conversion of ideal values to percentiles

Factors	Ideal Values	Percentile
Free time [13]	60	44.35%
Household space [14]	27.8709m^2	30.63%
Disposable income allocated to pets [15]	\$3215	73.01%
Animal experience	200 hours	53.97%
Temperature [16]	13-37°C (±12°C)	27.08%
Distance from Vet [12]	10km	52.87%

2.4 Readiness Score Function

The last step of this process, with the aim of determining the overall score for cat readiness, was the comparison of individual household graphs with the ideal cat graph. This was done by calculating the percentage of each individual graph which overlapped with the ideal graph, i.e.:

$$S_h = \frac{A_{so}}{A_{si}}$$

However, merely calculating the raw overlap posed some problems. Such a method would punish households exceeding expectations. Thus, additional area above the ideal was chosen to contribute to a household's overall score, effectively acting as additional area overlapping with the ideal shape. This allows a diverse range of households to succeed in pet ownership while maintaining the integrity of the model's intuition. It should be noted, however, that diminishing returns occur above the ideal thresholds, with extraordinarily little additional benefit to the pet occurring if a household has, for example, \$100 million rather than \$1 million disposable income. Due to this factor, the score increases from additional areas were weighted less than the standard scores. Due to this idea of diminishing returns, a logarithmic function was used to weight the excess area:

$$S_{ea} = \frac{1}{5}\log\left(\frac{A_{se}}{A_{si}} + \frac{1}{10}\right) + \frac{1}{5}$$

In this function, the ratio of excess area to ideal area forms the core of the logarithm's argument to communicate the idea of diminishing returns, while the horizontal and vertical translations ensure the function passes through the origin (as 0 excess area should equate to 0 extra score). The dilation then ensures that the excess area only adds a reasonable amount to the score. For example, using this function, a household with an excess area of 800 square units and an ideal shape area of 4400 square units would receive a bonus to their readiness score of 0.09, or 9%. The potential for these bonuses to elevate the score over 100% also presents an opportunity to rank applicants when there are more applicants with 100% score than the number of pets available in institutions. Thus, the final formula for the score was given as:

$$S_h = \frac{A_{so}}{A_{si}} + \frac{1}{5}\log\left(\frac{A_{se}}{A_{si}} + \frac{1}{10}\right) + \frac{1}{5}$$

This formula requires the areas of several irregularly shaped polygons, and so the Shoelace Formula (Gauss' Area Calculation Formula) was applied using the coordinates of vertices (x_i, y_i), where i ranges from 1 to n (number of factors) to determine the areas:

$$A = \frac{1}{2}\left|\sum_{i=1}^{n} x_i y_{i+1} - x_{i+1} y_i\right|$$

The radar charts were then generated through a Python code visualising the households against an ideal benchmark, facilitating a direct comparison through a visually intuitive manner. A geometric analysis was then conducted through the code, applying a geometric library to identify all intersection points to evaluating using the **Shoelace Formula**, the area of the ideal polygon (A_{si}), representing optimal readiness conditions, the household polygon (A_{sh}), the intersection area of the household and ideal polygons (A_{so}), and the household's excess area (A_{se}) (see Appendix 1 and Appendix 3 for automation).

2.5 Randomised Diverse Households

Thus, six example Australian households were generated to demonstrate the model:

Table 3 - Six diverse households, including attribute values and percentiles.

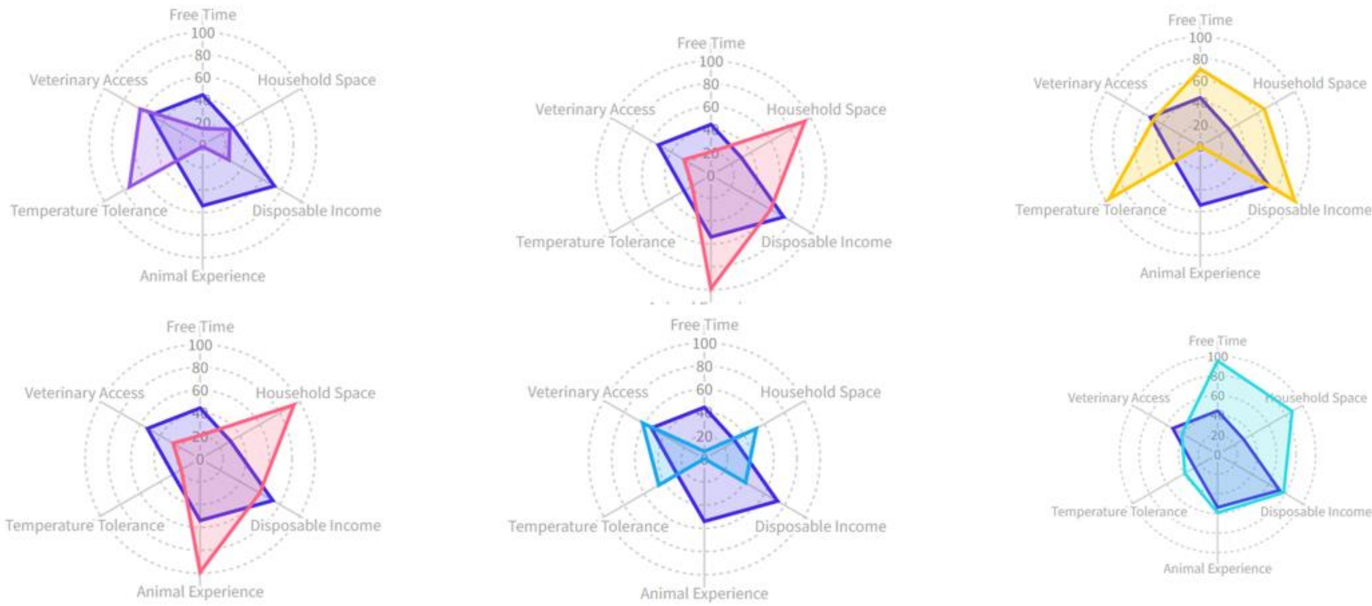
Context (Listed A-F)	F (mins)		H(m^2)		I_d (\$)		A_c (mths)		T (°C)		D_v (km)	
Family of 2 adults in Sydney CBD apartment	40	14.50%	26	27.58 %	1000	26.80 %	40	1.41 %	5	74.65 %	4	63.4 7%
2 Parents, 3 children on rural property, Alice Springs	45	20.38%	190	94.33 %	2500	59.93 %	380	99.12 %	14	18.87 %	25	27.1 3%
Dual-earning couple in suburban house, Rochedale, Brisbane	75	70.59%	50	67.86 %	6000	99.99 %	10	0.21 %	1	96.56 %	12	49.2 5%
2 parents, 2 children in social housing in Perth	55	35.56%	35	42.94 %	300	8.24%	160	32.57 %	2	92.29 %	9	54.6 7%

Young adult living in suburban unit, Hobart	30	6.41%	40	51.72 %	1600	41.06 %	0	0%	9	45.38 %	5	61.7 5%
One parent (animal instructor) & child in beach house at Cairns	100	95.22%	70	87.83 %	3500	77.80 %	210	59.37 %	10	38.59 %	16	42.0 6%

2.6 Testing Cat Model

Through comparison with the ideal cat shape using the method as developed above, Household A received a score of 39.08%, B a score of 87.51%, C a score of 78.31%, D a score of 64.73%, E a score of 36.94%, and F a score of 119.96%. Interestingly, F received a score of greater than 100%, due to the ability to gain extra score by exceeding expectations. For this example, the percentage score chosen as satisfactory was 75%. This score was used as the minimum requirement as 75% is the minimum value that indicates a strong correlation between variables in statistics. However, in practice, the percentage score requirement for pet readiness may be altered as the IMMC-A sees fit, depending on the level of pet readiness certainty desired.

Figure 3 - Radar charts of cats overlayed by household stats (note the overlap)



Based on this requirement, Households B, C, and F were found to fulfil the 75% threshold for pet readiness. This demonstrates the diverse ways by which a household can achieve pet preparedness. Some households, such as F, can achieve pet preparedness through moderate strength in all areas, while some households, such as C, can do so with great strength in fewer areas to compensate for weakness in others.

Notably, the scores with the highest ideal value (the scores most rewarded for high fulfilment) completely depend on the animal-specific data, thus providing a prominent level of generalisability. Additionally, the nature of the model's weighting - rewarding households which attempt to match all ideal scores - indicated that a balance in all factors, rather than a great excess in one or two, was the best way to achieve pet readiness.

Evidence of this can be seen in F's much greater success than C, with C having great strength in some factors to compensate for others, while F had sufficient, but not outstanding, strength in all. That is not to say, however, that a household which may have an excess in one factor but is weaker in the others cannot achieve pet readiness, as shown by C, and this is one of the greatest strengths of the model: the flexibility of the criteria.

2.7 Total Pet Readiness

Next, the model was used to determine the total pet readiness of households across three regions. It was noted that the current model, while greatly successful for individual cases, was practically impossible to automate across all households of a given country or region. Therefore, one key simplification was made. To translate the model into a much wider case, as it was considered that as the success mark of 75% provided a useful benchmark for universal readiness, perhaps this could be translated into a factor-by-factor analysis. In a proposed new system, a household's pet-readiness would be assured if they reached 75% of the ideal mark in each factor. However, it was then considered that many households would have one factor which would most likely be below the 75% mark, as can be seen in the example six households above - only Household F, which is an outlier in terms of overall strength, would pass.

Therefore, the new requirement for pet readiness was determined to be a score of 75% of the ideal or more, in each household's 5 best factors. This system would also result in the removal of excess points, as it was considered that below 75% of the ideal mark in any two factors would present too great of a liability to the household's pet-readiness to allow them to pass, no matter their success in other factors.

Table 44 - Adjusted ideal values and percentiles for population modelling.

Factors	New Threshold Values	Percentile
Free time	45	20.38%
Household space	20.903175m^2	19.84%
Disposable income allocated to pets	$2411.25	58.19%
Animal experience	150 hours	27.75%
Temperature	±16°C	13.78%
Distance from Vet	13.3km	46.90%

The three regions chosen to be investigated were Singapore, New York, and Copenhagen. It was first observed that within each region, the climate remained essentially constant, with minimal internal deviations. Thus, the climates were set for each region at 13.2°C for New York [17] (28.09%), 26.5°C for Singapore [18] (94.54%) and 9.3°C for Copenhagen [19] (12.60%). With this set, the determination of overall pet readiness was essentially reduced to a probability problem, and it is this problem which must be solved to find pet readiness percentage.

Firstly, it was found that for New York and Singapore, the climate percentiles both exceeded the 75% mark, and thus their excluded factor would come from the group of other five factors. The probability can be considered as the sum across all factors of the probability of a given factor being the weakest multiplied by the probability of fulfilling all other factors given the failure of one.

$$P = \sum_{i=1}^{5} \Pr(\text{Factor } i \text{ fails}) \times \Pr(\text{All other factors succeed})$$

By definition, this can be expressed as a matrix multiplication for ease of computation and visual intuition:

$$P = (0.2038 \quad 0.1984 \quad 0.5819 \quad 0.2775 \quad 0.4690) \begin{pmatrix} 0.8016 \times 0.4181 \times 0.7225 \times 0.531 \\ 0.7962 \times 0.4181 \times 0.7225 \times 0.531 \\ 0.7962 \times 0.8016 \times 0.7225 \times 0.531 \\ 0.7962 \times 0.8016 \times 0.4181 \times 0.531 \\ 0.7962 \times 0.8016 \times 0.4181 \times 0.7225 \end{pmatrix}$$

$$P = 0.3239$$

For the case of Copenhagen, contrastingly, the climate failed to meet the minimum threshold, and so all other factors must be above the threshold to be pet ready.

$$P = 0.7962 \times 0.8016 \times 0.4181 \times 0.7225 \times 0.531$$
$$P = 0.1025$$

These probabilities of being cat-ready can then be multiplied by the number of households per region to give the number of cat-ready households per region. The number of households was found to be 7,604,523 for New York [20], 1,399,600 for Singapore [21], and 343,498 for Copenhagen [22]. Therefore, multiplying by the probability of cat-readiness for each city, it was found that the current number of cat-ready houses in New York was 2,463,105, 453,330 in Singapore, and 35,208 in Copenhagen.

3.0 Generalisation of Model

3.1 Scoring for Different Pets

The model was then generalised to incorporate multiple additional animals, as this greatly expands the overall usefulness of the model. Therefore, by nature of the model's construction, this generalisation only required the acquisition of additional data. Similarly to the ideal values for the cat, these values were collected [1],[23]-[38] and transformed into percentiles through the statistical distributions outlined earlier.

Table 5 - Ideal values and percentiles of factors for chosen pets.

Factors	Dogs		Goldfish		Horse		Parrot	
	Ideal	Percentile	Ideal	Percentile	Ideal	Percentile	Ideal	Percentile
Free time	90	88.89%	4	0.01%	85	84.02%	100	95.22%
Household space	37.16	46.75%	0.124	0.01%	405.0573	94.34%	5	2.99%

Disposable income allocated	\$3218	73.06%	\$250	6.83%	\$4900	98.28%	\$1500	38.78%
Animal experience	150	27.75%	12	0.25%	240	74.15%	220	64.59%
Temperature	±16	13.78%	±3	87.18%	±7	60.09%	±7	60.09%
Distance from Vet	10	52.87%	20	35.13%	25	27.13%	14	45.64%

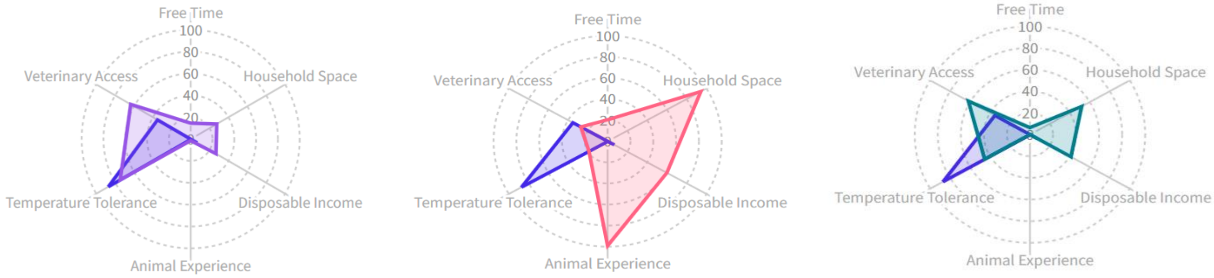
Due to the high generalisability of the model, no notable change was required other than simply automating the Shoelace formula calculations through Python (Appendix 3). The following scores were determined for the six households introduced earlier (see Appendix 4 for radar charts).

Table 6 5 - Scores of households for chosen pets.

Households	Score			
	Dog	Goldfish	Horse	Parrot
A	29.66%	118.93%	16.88%	38.18%
B	67.76%	51.34%	49.49%	57.82%
C	90.26%	134.78%	54.33%	67.83%
D	48.53%	129.64%	30.66%	67.12%
E	30.42%	88.86%	15.97%	30.35%
F	114.34%	89.54%	81.38%	100.32%

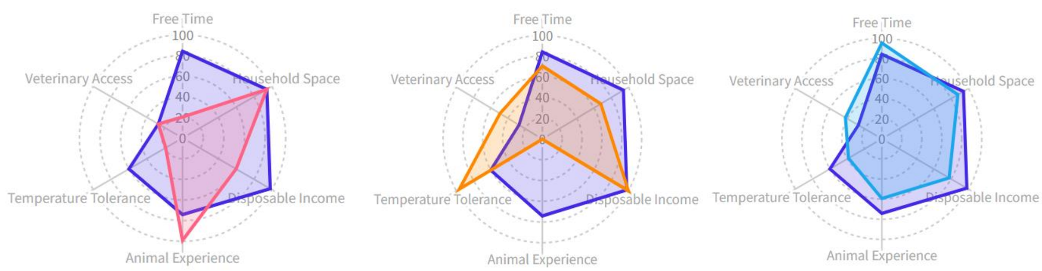
Which meant that, using the previously determined success mark of 75%, Households C and F qualified for dogs, Households A, C, D, E, and F qualified for goldfish, Household F qualified for horse, and Household F qualified for parrots. The first key result to note was the ability of households which failed the pet-readiness test for cats to qualify for other animals which required a different balance of results. For example, though Households A and E showed very weak results when evaluated against the cat shape, both households achieved extraordinarily strong results when tested against the goldfish, due to their strength in areas that the cat largely disregarded but the goldfish favoured. This highlights the effectiveness of the model: households which are determined unsuitable for one particular pet can be ready for a different one depending on their particular strengths and the animal's particular needs. Similarly, a strong household when measured against one animal, such as Household B, can fail when measured against another. Collectively, these results indicate a strong ability of the model to make reasonable and discerning decisions (full result set in Appendix 4).

Figure 4 - Above from left to right: Households A, B, and E compared against the Ideal Goldfish



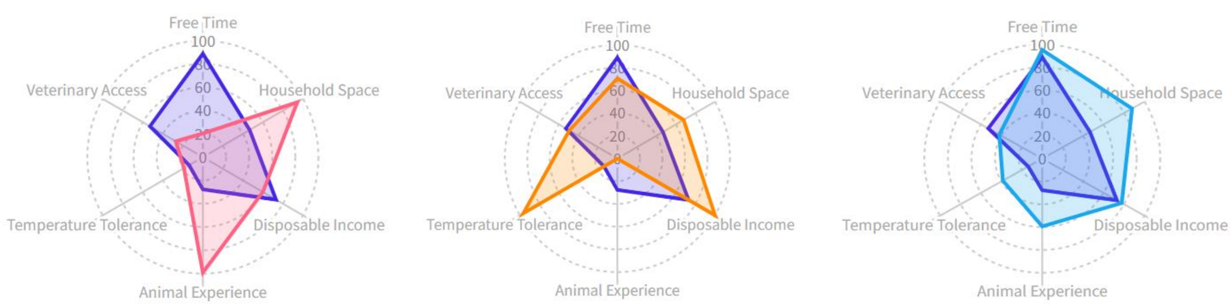
The second result worthy of discussion came from the comparison of the six households to the horse. Logically, a horse is a highly demanding pet, and so very few households should meet its requirements. For example, Household B had high results in some of the factors required by the horse, but as the ideal horse shape had multiple high factors, Household B failed to meet the 75% threshold. This was similarly the case for Household C, which had generally strong results but failed due to its lack of animal experience. However, Household F, clearly the strongest household overall, did manage to reach the 75% threshold, due to its strength across multiple factors, while its weaknesses were factors not highly desired by the horse. This demonstrated again the model's ability to make reasonable judgements, assigning a clearly difficult animal only to the household which convincingly seemed prepared for it.

Figure 5 - Above from left to right: Households B, C, and F compared against the Ideal Horse



The results obtained for the dog were remarkably similar to those for the cat, given the similarity of their two ideal shapes. However, Household B failed to meet the requirements for dog readiness, due to its more cat-friendly statistics. The two other households which passed the threshold for cat readiness additionally did so for dog readiness, due to their slightly different distribution of strong factors. This indicates the model has an ability to detect nuances between different spreads of factors to best detect pet readiness.

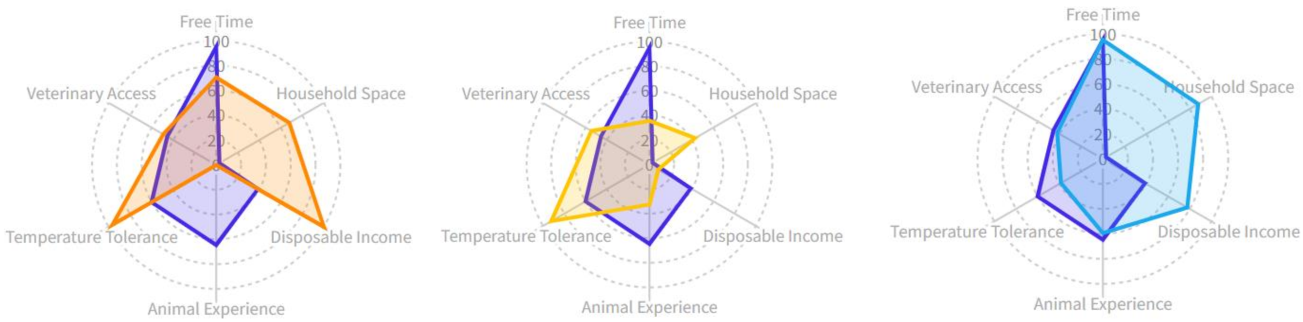
Figure 6 - Above from left to right: Households B, C, and F compared against the Ideal Dog



Lastly, the parrot, another reasonably demanding animal, had comparable results to the horse, in that only Household F succeeded in reaching the threshold of 75%. Unlike the horse, contrastingly, Households C and

D both achieved scores of around 67%, nearly enough to qualify for parrot readiness. This demonstrates another strength of the model: the ability to rank households based on degree of failure or success to provide alternatives and options to the IMMC-A.

Figure 7 - Above from left to right: Households C, D, and F compared against the Ideal Parrot



3.2 Assessing ISC Using Matrices

It was then considered that many households have multiple pets of different species, which must be accounted for within the model. It was thus decided to introduce an interspecies compatibility factor, which would be considered in conjunction with pet-household readiness to determine an overall readiness. With the aim of adding interspecies compatibility as an additional factor to add to the radar charts, the first step was the construction of pet-by-pet compatibilities.

Firstly, it was decided that similar energy levels should lead to a higher compatibility score, as two animals with similar energy levels will be more suited to each other behaviourally. Similarly, for bite force, animals with similar bite forces would be more suited to each other, bite force being a general indicator of physicality. As for mass, animals with similar masses would logically be more suited for each other, and so should receive a higher compatibility score.

The final two contributing elements to the interspecies compatibility factor were effectively yes-or-no concerns. If the two animals occupy different environments, the interspecies compatibility factor should automatically be near the maximum possible, as very limited possibility exists of the two animals harming each other. Similarly, if a predator-prey relationship exists between the two animals and they occupy the same environment, the interspecies compatibility factor should be set automatically to the minimum, as the risk of harm to one or more of the pets is too great to allow, and this takes precedence over all elements. A method was then found to quantify the similarity between two metrics, using the following equation.

$$S = \frac{x_{max} - \bar{x}}{\bar{x}}$$

This equation finds, as a percentage of the mean, the distance from the mean to the maximum value between the two. This is effectively a calculation of similarity, as the smaller the distance from the mean to the maximum value, the more similar the two values are. This equation can be used to find the similarity for the energy levels, bite force, and mass [39]-[52].

Table 7 - Attributes of selected pets

Pet factors	Cat	Dog	Goldfish	Horse	Parrot
Energy Levels (Cal)	300	1225	2.5	2000	550
Bite Force (PSI)	70	406	2	500	1150
Mass (kg)	4.54	7.46	0.2	500	0.4
Environmental Interference	Land	Land	Water	Land	Air
Predator-Prey (0 or 1)	0	0	1	0	1

The above data resulted in the following pet-pet compatibility scores ($0 < S_p < 1$):

Table 8 - Interspecies compatibility (1st order)

	Cat	Dog	Goldfish	Horse	Parrot
Cat	0.875	0.736	0.789	0.506	0
Dog	0.736	0.875	0.767	0.796	0
Goldfish	0.789	0.767	0.875	1	0.920
Horse	0.506	0.796	1	0.875	1
Parrot	0	0	0.920	1	0.875

A matrix method was then used to calculate a percentage score for each household as follows:

Firstly, the initial pet status (number and type) of the household was considered and formed into a n-x-n matrix (n being number of pets), with A_{ij} equal to the compatibility between pets i and j for this matrix $\boldsymbol{A}$. The new pet was then added to the matrix as the $(n+1)^{th}$ row and column.

Then, taking inspiration from the method of dominance matrices, for this matrix $\boldsymbol{A}$, the result $\boldsymbol{A} + \frac{1}{2}\boldsymbol{A}^2 + \frac{1}{3}\boldsymbol{A}^3$ was found. Squaring and cubing the matrices allows to incorporate second- and third-order pet compatibilities, or in other words the compatibility of a pet with other pets based on an intermediate pet's compatibility with both. This provides a more detailed indication of pets' compatibilities with each other.

With this sum found, it would be post-multiplied by a column matrix of ones and then pre-multiplied by a row matrix of ones, to find the total of all compatibilities. This total would then be divided by the maximum possible such score, which would be the result of this process with a compatibility matrix of all ones.

EXAMPLE - Family C has a goldfish and cat, now they want a dog. Consider columns and rows 1, 2, and 3 as Goldfish, Cat, and Dog, in that order.

$$\boldsymbol{D_c} = \begin{pmatrix} 0.875 & 0.789 & 0.767 \\ 0.789 & 0.875 & 0.736 \\ 0.767 & 0.736 & 0.875 \end{pmatrix}$$

$$\boldsymbol{D}_{\mathbf{max}} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$$\boldsymbol{D_C} + \frac{1}{2}\boldsymbol{D_c}^2 + \frac{1}{3}\boldsymbol{D_c}^3 = \begin{pmatrix} 3.44 & 3.32 & 3.27 \\ 3.32 & 3.38 & 3.21 \\ 3.27 & 3.21 & 3.33 \end{pmatrix}$$

$$\boldsymbol{D}_{\mathbf{Max}} + \frac{1}{2}\boldsymbol{D}_{\mathbf{Max}}^2 + \frac{1}{3}\boldsymbol{D}_{\mathbf{Max}}^3 = \begin{pmatrix} 5 & 5 & 5 \\ 5 & 5 & 5 \\ 5 & 5 & 5 \end{pmatrix}$$

Pre- and post-multiplication by row and column matrices of 1s then found the example and maximum scores.

$$D_{\mathrm{C\ score}} = 29.75$$

$$D_{\mathrm{Max\ score}} = 49.5$$

$$S_p = \frac{D_{\mathrm{C\ score}}}{D_{\mathrm{Max\ score}}} = 0.603$$

This score becomes a given household's score for the new seventh radar chart factor (interspecies compatibility). The only other key element of accounting for multiple pets is then to ensure that the factor of disposable income assigned for other pets is subtracted from the household's amount when considering a new pet, and free time and household space are subtracted with a multiplier (as these factors can be shared between pets to some extent) – which is addressed in the section 4. Thus, the model is completely generalised to any number and type of pets. The example above demonstrates how the model can be adapted for these conditions.

4.0 Modelling Future Trends

The task of modelling future pet ownership was observed to be well-suited to the technique of differential equations. By modelling the change in pets as a function of parameters easily found from above work, the general trend in pet ownership could be easily found.

We first let change in pets over time be $\frac{dP}{dt}$, with pet population $P(t)$.

This depends on several factors, namely proportion of pet-readiness, proportion of pet-willingness, human/household population, and pet loss rate. In most simple form:

$$\frac{dP}{dt} = increase\ in\ pets\ +\ decrease\ in\ pets$$

First note that the model generalisation from above is being used. This means that each household requires above 75% in their best five factors from the standard six, and additionally above 50% in interspecies compatibility. Interspecies compatibility is required as it is essential for pet wellbeing and therefore its threshold must be met. 50% was chosen for this factor rather than 75% because third order compatibilities were counted, meaning that achieving a high percentage is nearly impossible (as fractions of 1 degenerate rapidly to 0 with every order added). Now several factors must be considered. Firstly, as it was assumed for future predictions that households will only get pets of the same type (due to trends in pet species/demand based on previous pets being practically impossible to predict), the interspecies compatibility score will always remain above 50%. This both logically and mathematically follows (pet compatibility for the 2nd pet of the same kind is 76.02%, 3rd pet is 73.33%, and 6th is 70.01%). Therefore, the only limiting factor preventing households from continuing to get pets is willingness and resources.

Resources were firstly addressed with a **Generalised Readiness Function**. Building on the generalisation of the model from earlier, which gave an output of readiness probability, this function will output the readiness percentage of the population for each number of pets (2nd pet, 3rd pet, etc.). Logically, the resource demand increases with each pet. Thus, the percentage of the ideal required in a household's best five factors should increase with each pet. As many resources can be shared between two pets of the same kind, two pets should require less than 150% but more than 100% of the ideal for one pet across the 5 best factors, with diminishing requirements as more pets are added. Thus, $R(n)$ will output the percentage of the population meeting $\left(50 + \frac{50n}{n+1}\right)$% of the ideal value for their five best values (using the model generalisation from 2.7).

Willingness will be similarly addressed, though the mathematics is less arduous. A statistical investigation reveals that around 58.65% of people own one pet, with 8.97% owning two, and 2.07% owning three [53][54]. These numbers were graphed, and an exponentially decaying function fitted with regression (to match the data's obvious nature). This gave willingness to get n pets, $W(n)$, as:

$$W(n) = e^{-1.8555n+1.32165}$$

Now let $P_n(t)$ be the population of households with n pets at time t. Additionally, let the loss rate of pets per year be L. Thus, for any P_n,

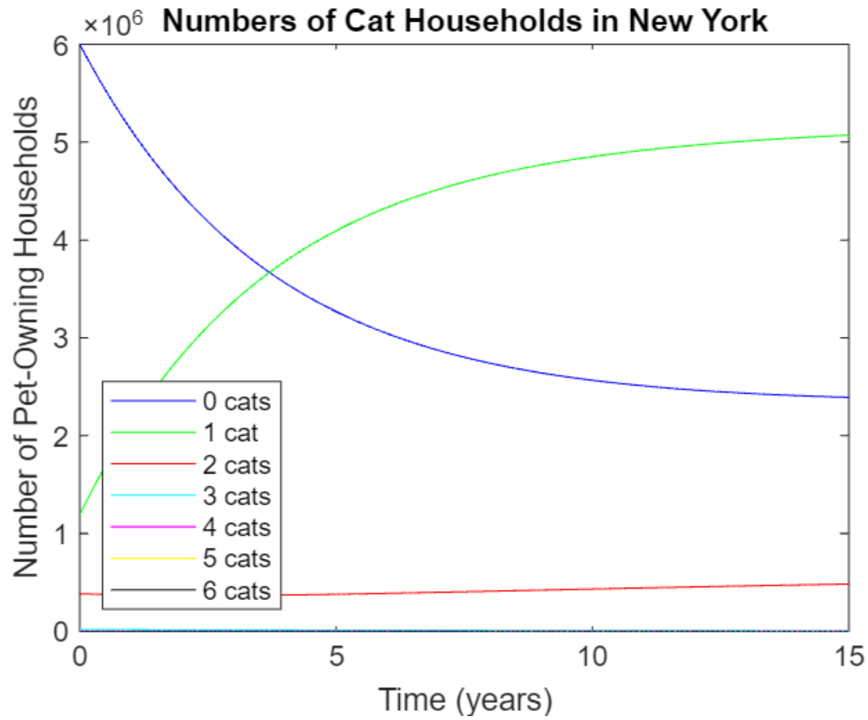
$$\frac{dP_n}{dt} = R(n) \times W(n) \times P_{n-1}(t) + L \times P_{n+1}(t) - L \times P_n(t) \; - \; R(n+1) * W(n+1) * P_n(t)$$

Observe that each P_n gains households based on the households with one more pet who have lost a pet. Each P_n also loses households for each one added to P_{n+1}, as when a household gains a pet, they no longer count towards the number of households with their previous number of pets. Additionally, $P_0(t)$ should be equal to the current number of households without a pet (including human population growth). To model household population growth, the traditional logistic growth model was used, to provide an accurate model of available households increasing. This did require the collection of additional data (k, growth factor, and C, carrying capacity) but this data was readily available. Finally, note that beyond six pets per household, the number of households becomes statistically insignificant, at less than 0.000001% of the population. Therefore, the system will be considered up to $P_6(t)$. The system of ODEs was then expressed in vector form.

$$\frac{d}{dt}\begin{pmatrix} P_0(t) \\ P_1(t) \\ P_2(t) \\ P_3(t) \\ P_4(t) \\ P_5(t) \\ P_6(t) \end{pmatrix} = \begin{pmatrix} kP_0(1-\frac{P_0}{C}) + L \times P_1(t) \; - R(1) \times W(1) \times P_0(t) \\ R(1) \times W(1) \times P_0(t) + L \times P_2(t) - L \times P_1(t) - R(2) \times W(2) \times P_1(t) \\ R(2) \times W(2) \times P_1(t) + L \times P_3(t) - L \times P_2(t) - R(3) \times W(3) \times P_2(t) \\ R(3) \times W(3) \times P_2(t) + L \times P_4(t) - L \times P_3(t) - R(4) \times W(4) \times P_3(t) \\ R(4) \times W(4) \times P_3(t) + L \times P_5(t) - L \times P_4(t) - R(5) \times W(5) \times P_4(t) \\ R(5) \times W(5) \times P_4(t) + L \times P_6(t) - L \times P_5(t) - R(6) \times W(6) \times P_5(t) \\ R(6) \times W(6) \times P_5(t) - L \times P_6(t) \end{pmatrix}$$

This system of ODEs, while unsolvable by hand, can be solved using any number of numerical solvers. For this paper, MATLAB was used to generate a solution (Appendix 5), as it has efficient built-in solvers for non-stiff ODE systems such as this one. After solving the system numerically using MATLAB, a number of solution curves were generated using initial conditions gathered from data. While many of the actual values varied by animal and region, in general the prediction of pet numbers followed the following pattern (here shown as future cat populations in New York).

Figure 8 - Predicted population of cats in New York.



The number of pet-less households decreases quickly, corresponding to a sharp rise in households with one pet. The rate of change of both slows as the number of households available to get a pet decrease, and the proportion begins to fit the "natural" proportion dictated by the readiness and willingness proportions. The number of households with two pets steadily rises, though the number of households with three or more pets usually decreases, given that the loss rate tends to outpace the percentage of the population able and willing to acquire a third, fourth, or even fifth pet. As an example, in 5, 10, and 15 years, the number of single-cat households in New York was projected at 4.1, 4.9, and 5.1 million, the number of double-cat households at 0.37, 0.43, and 0.48 million, and the number of triple-cat households at 13719, 947, and then 739. These numbers serve as an example to communicate the key idea of the future projections. The complete list of projections and graphs can be found in Appendix 6.

5.0 Sensitivity Analysis of Key Model Aspects

There are two aspects that can be manipulated to conduct a sensitivity analysis of the suitability of a pet to a household used throughout this paper: accuracy of values input by a household and multiplier of readiness benchmarks used for model generalisation. Firstly, it is unlikely for data put in by households to be completely accurate, and while this was assumed to be the case, a sensitivity analysis was conducted on the score output for a household compared against the ideal "shape" of a pet in order to investigate the effect of errors in real life. Using the example of Family D against a cat, being close to the pass mark and yet failing with a value of 64.73%, random input error was simulated in sets of 200 households, varying maximum error from 5% to 30% in intervals of 5% (Appendix 7). Using the normal distribution to find the mean & standard deviation of each set of simulated households, the number of erroneous successes and error margin was calculated for respective input error magnitudes (see appendix 8 for distribution & raw data and 7 for code).

Figure 9 – Score distribution for various input errors.

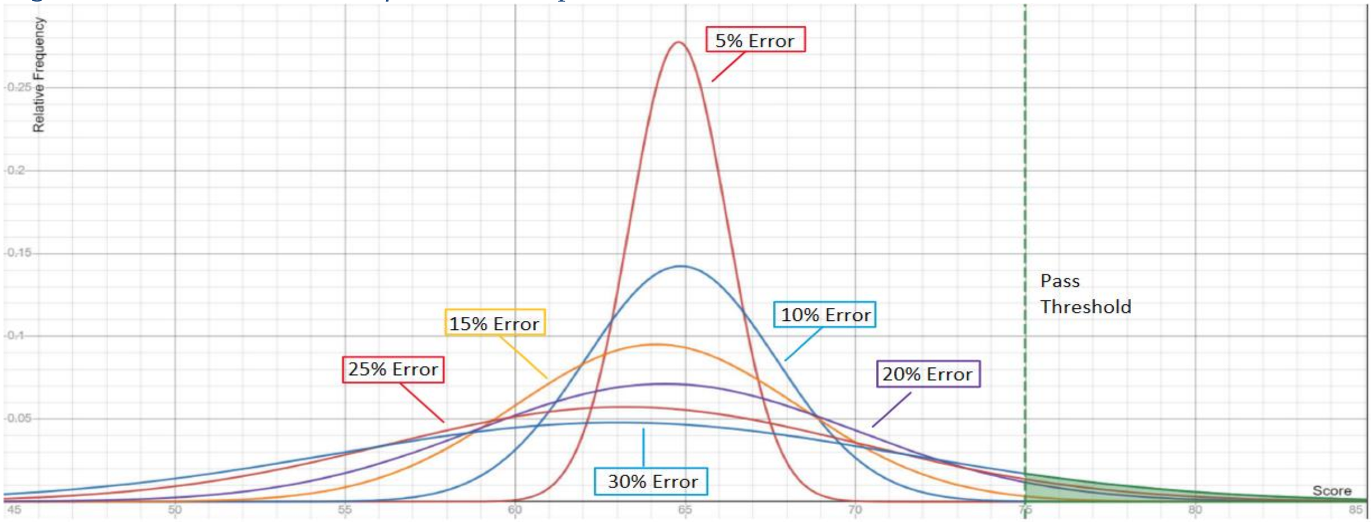
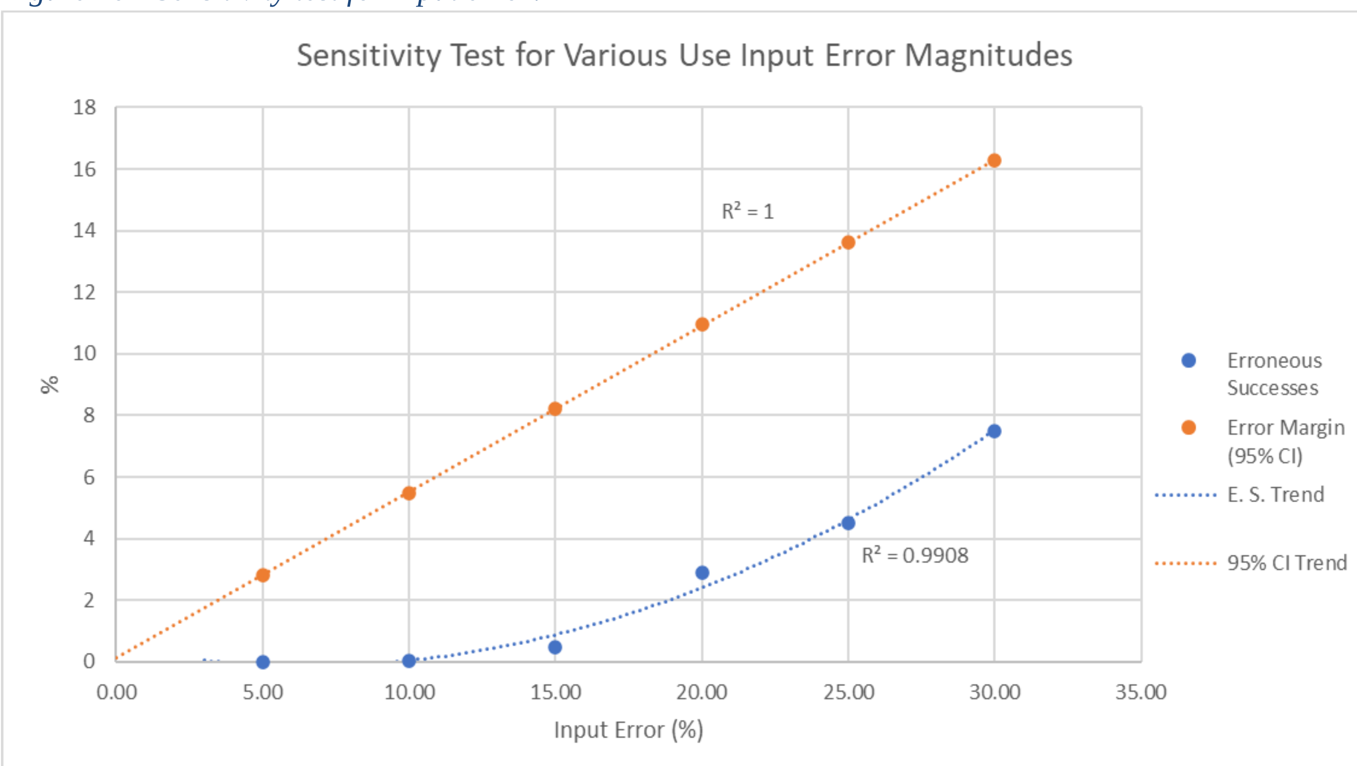


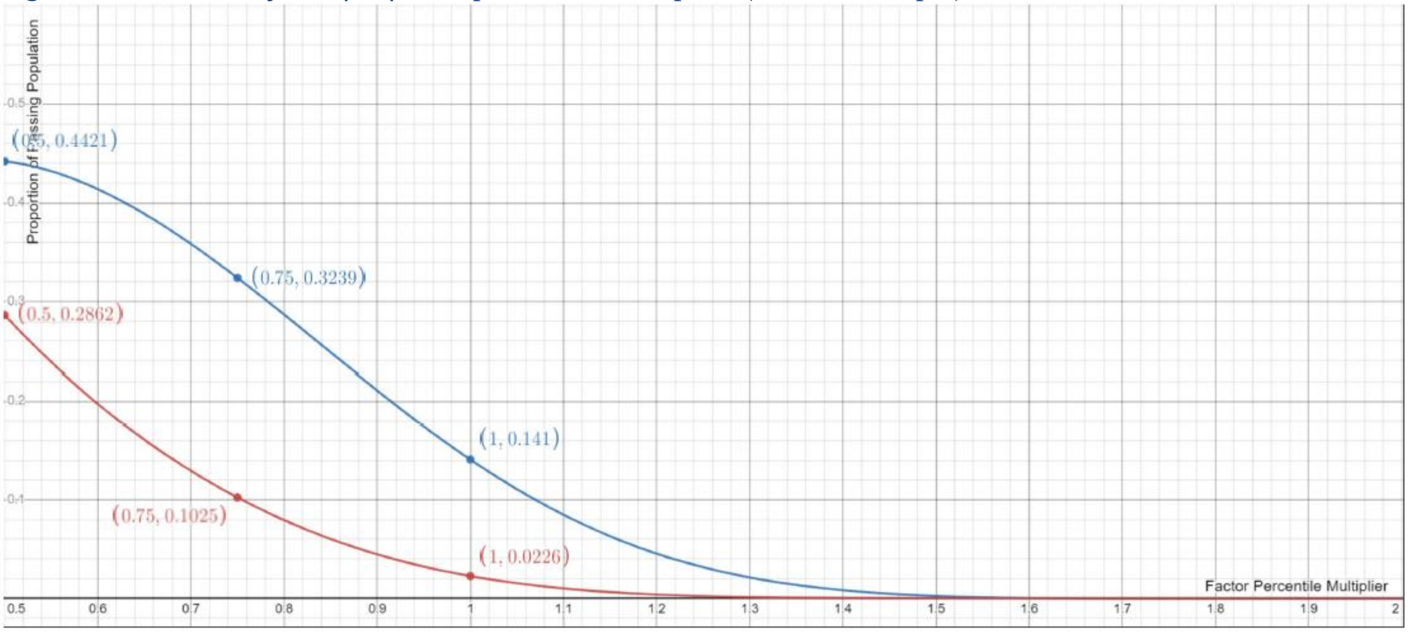
Figure 10 - Sensitivity test for input error.



Graph 10 shows that input error has a positive linear correlation with the error margin of scores generated, with the margin being about half of the input error in the 95th confidence interval. Additionally, this confidence interval does not reach the success benchmark of 75% until an extremely high input error of 30% - indicating that the model is reasonably consistent in determining results. This is confirmed by the observation that the occurrences of erroneous successes remain notably low until ~20% error, where it then increases linearly. With a reasonable error for most households at ~10-15%, its clear that the developed method of compatibility is relatively insusceptible to inaccuracies in input, and as such is robust in handling potential common changes to household situations (as these changes are small).

6.0 Sensitivity Analysis of Percentile Multiplier

Figure 11 - Sensitivity test for factor percentile multiplier (cats as example)



In this paper, it was decided that an overall score of 75% (multiplier = 0.75) would be sufficient for readiness, though the task stated that pet shops and animal shelters have the flexibility to choose any score they desire as a pass mark. Therefore, a sensitivity analysis is required to understand the reasonableness of a benchmark chosen for the New York-Singapore (P) and Copenhagen (C) function, respectively.

Plotting the percentage of households that are ready to own a pet (cats in this graph), as created in part 1b, shows that as the percentile multiplier (score requirement) increases, the proportion of ready households decreases at a declining rate. This is expected as there should be less households being able to cater for the increasingly higher factor values. Interestingly, beyond a score of 100%, there is still a non-zero proportion of households ready to cater for such needs, which is representative of household samples such as household F, which had a score over 100%. Additionally, these functions illustrate the mean probability of success for any random households for the 3 analysed regions, and as such serves as a guideline for selecting desired pass rates.

7.0 Evaluation

7.1 Strengths

- The model was user-friendly, requiring seven inputs which should all be readily available to households.
- The model is generalisable to all pets, household situations and most regions, due to the common, percentile-derived scale.
- The model allowed for a semi-realistic and reasonable application of score readiness so that the number of pet households in the future could be calculated. Differential equations are a traditional and effective method of mapping changing population dynamics such as this one.
- **Radar charts** facilitated an intuitive and immediate understanding of complex data, while geometric analysis through the **Shoelace Formula** then applied mathematical rigour to the readiness metric.

7.2 Weaknesses

- Data does not accurately reflect the deviations within a species, which can modify the percentiles of its attributes. This effect can propagate, affecting the actual readiness of the household to own their intended pet and resulting in some inaccuracies within population projection models.
- Trend development does not consider the dynamic situation of households in the population, which will impact the rate and number of households ready to own pets in prediction models. However, modelling changing household metrics across multiple regions is resource-intensive and lacked quantifiable data.
- Certain distributions only reflect the global situation and not region-specific, e.g. the distribution of disposable income is different in third-world countries compared to the global distribution. This can be circumvented by changing distributions to match regional data in countries with significant deviations.
- Population projection models did not account for households with combinations of different pets, which can impose errors associated with the growth rate of certain pets. However, accounting for such is nearly impossible, and the current model still provides a good indication of future pet demographics.
- While the model attempts to account for numerous factors influencing pet readiness, it inevitably simplifies the complex interactions between factors, especially qualitative ones (e.g. emotional and behavioural compatibility), which may not capture the full dynamics of household-pet relationships.

7.3 Conclusion

The objective of this paper was to develop an adjustable, quantitative decision metric that determined the suitability of a pet for a household. Through radar charts, the model considers six general factors from a household's living standards to determine this readiness as a numerical value. Using matrices and these six factors, the number of households in any region prepared to own a pet was also obtained. To include the factor of compatibility of a pet with other household pets, an ISC score was introduced, again using matrices. Lastly, using differential equations to model the growth or decline of households with different pets, the model predicted the future of pet ownership over the next 5,10 and 15 years. A sensitivity analysis also revealed that marginal error in household inputs barely affected the percentage of possible households prepared for the ownership of a pet, with the readiness score function being valid for any pet shop allowing a passing score of 50%. These results demonstrated the generalisability of the reasonable approach generated for any household considering buying a pet.

8.0 References

[1] Animal Medicines Australia, “Pets in Australia: A National Survey of Pets and People,” 2022. Available : https://animalmedicinesaustralia.org.au/wp-content/uploads/2022/11/AMAU008-Pet-Ownership22-Report_v1.6_WEB.pdf

[2] Flourish, “Radar Chart,” *Flourish*. https://app.flourish.studio/@flourish/radar (accessed Feb. 20, 2024).

[3] Australian Bureau of Statistics, “Special Feature: Household Pets,” *www.abs.gov.au*, Jun. 20, 1995. https://www.abs.gov.au/Ausstats/abs@.nsf/7d12b0f6763c78caca257061001cc588/5ef8016f420622a3ca2570ec00753524

[4] Australian Bureau of Statistics, “How Australians Use Their Time,” *www.abs.gov.au*, Oct. 07, 2022. https://www.abs.gov.au/statistics/people/people-and-communities/how-australians-use-their-time/latest-release

[5] Organisation for Economic Co-operation and Development, “Time Use,” *Oecd.org*, 2019. https://stats.oecd.org/Index.aspx?datasetcode=TIME_USE

[6] Entranze, “Average Floor Area per Capita,” *entranze.enerdata.net*. https://entranze.enerdata.net/ (accessed Feb. 22, 2024).

[7] “How Much Living Space Does the Average Household Have in Japan?” *https://resources.realestate.co.jp*, Aug. 12, 2017. https://resources.realestate.co.jp/living/how-much-living-space-does-the-average-household-have-in-japan/ (accessed Feb. 21, 2024).

[8] M. Megna, “Pet Ownership Statistics and Facts in 2023,” *www.forbes.com*, Apr. 10, 2023. https://www.forbes.com/advisor/pet-insurance/pet-ownership-statistics/#:~:text=As%20of%202022%2C%2044.5%25%20of

[9] Queensland Government, “Duty of Care for Animals,” *www.business.qld.gov.au*, Jun. 01, 2011. https://www.business.qld.gov.au/industries/farms-fishing-forestry/agriculture/animal/health/welfare/duty#:~:text=If%20you%20have%20a%20duty (accessed Feb. 21, 2024).

[10] V. Freitas *et al.*, “Temperature Tolerance and energetics: a Dynamic Energy budget-based Comparison of North Atlantic Marine Species,” *Philosophical Transactions of the Royal Society B: Biological Sciences*, vol. 365, no. 1557, pp. 3553–3565, Nov. 2010, doi: https://doi.org/10.1098/rstb.2010.0049.

[11] “List of Countries by Average Yearly Temperature,” *Wikipedia*, Apr. 02, 2019. https://en.wikipedia.org/wiki/List_of_countries_by_average_yearly_temperature

[12] S. James, “How Far Is Too Far When Choosing a vet?” *Vet Help Direct*, Nov. 02, 2022. https://vethelpdirect.com/vetblog/2022/11/02/how-far-is-too-far-when-choosing-a-vet/ (accessed Feb. 21, 2024).

[13] The Cat Protection Society of NSW Ltd, “Cat Protection Society of NSW,” *https://catprotection.org.au*. https://catprotection.org.au/cat-care-factsheets/factsheet-indoor-cats/ (accessed Feb. 22, 2024).

[14] C. O'Brien, "How Much Space Does a Cat Need?" *Hill's Pet Nutrition*, Apr. 16, 2020. https://www.hillspet.com/cat-care/new-pet-parent/how-much-space-does-a-cat-need#:~:text=It%20might%20be%20hard%20to (accessed Feb. 21, 2024).

[15] Australian Government and Australian Securities & Investments Commission, "Getting a Pet," *moneysmart.gov.au*. https://moneysmart.gov.au/family-and-relationships/getting-a-pet#:~:text=In%20the%20first%20year%2C%20a (accessed Feb. 21, 2024).

[16] C. O'Brien, "What Temperature Do Cats Like?" *Hill's Pet Nutrition*, Jan. 03, 2023. https://www.hillspet.com/cat-care/routine-care/what-temperature-do-cats-like#:~:text=Any%20temperature%20over%20100%20F (accessed Feb. 21, 2024).

[17] Weather Spark, "Average Weather in New York City, New York, United States, Year-round - Weather Spark," *Weatherspark.com*, 2019. https://weatherspark.com/y/23912/Average-Weather-in-New-York-City-New-York-United-States-Year-Round

[18] Holiday Weather, "Singapore, Singapore - Average Annual Weather," *Holiday-weather.com*, 2020. https://www.holiday-weather.com/singapore/averages/ (accessed Feb. 22, 2024).

[19] Holiday Weather, "Copenhagen, Denmark - Average Annual Weather," *www.holiday-weather.com*, 2020. https://www.holiday-weather.com/copenhagen/averages/

[20] United States Census Bureau, "U.S. Census Bureau QuickFacts: New York," *www.census.gov*, Jul. 01, 2023. https://www.census.gov/quickfacts/fact/table/NY/PST045223

[21] A. Lim, "13+ Housing & Household Statistics in Singapore (2023)," *SmartWealth Singapore*, Feb. 22, 2024. https://smartwealth.sg/housing-household-statistics-singapore/#:~:text=is%20so%20important.- (accessed Feb. 23, 2024).

[22] Statistics Denmark, "Households and Families," *www.dst.dk*. https://www.dst.dk/en/Statistik/emner/borgere/husstande-familier-og-boern/husstande-og-familier (accessed Feb. 22, 2024).

[23] "How Much Exercise Does Your Dog Actually Need?" *https://animalsmatter.com/*, 2024. https://animalsmatter.com/blogs/news/how-much-exercise-does-your-dog-actually-need

[24] "How Many Dogs Do You Have and How Big Is Your house?" *Golden Retriever Dog Forums*, May 09, 2008. https://www.goldenretrieverforum.com/threads/how-many-dogs-do-you-have-and-how-big-is-your-house.35192/ (accessed Feb. 23, 2024).

[25] Animal Care Australia, "Cost of Owning a Pet," *https://www.animalcareaustralia.org.au*, 2019. https://www.animalcareaustralia.org.au/structure/responsible-pet-ownership/cost-of-owning-a-pet/#:~:text=Cats%20and%20dogs%20are%20a (accessed Feb. 22, 2024).

[26] M. Jordan, A. Bauer, J. Stella, and C. Croney, "Temperature Requirements for Dogs," Oct. 2016. Available : https://www.extension.purdue.edu/extmedia/va/va-16-w.pdf

[27] RSPCA, "What Should I Feed My Pet goldfish?" *RSPCA Knowledgebase*, Apr. 30, 2019. https://kb.rspca.org.au/knowledge-base/what-should-i-feed-my-pet-goldfish/#:~:text=Feed%202%2D3%20times%20daily

[28] Aquarium Co-op, "Care Guide for Fancy Goldfish," *Aquarium Co-Op*. https://www.aquariumcoop.com/blogs/aquarium/care-guide-for-fancy-goldfish#:~:text=What%20Size%20Tank%20Do%20Fancy (accessed Feb. 22, 2024).

[29] A. Jones, "How Much Do Goldfish cost?" *The Goldfish Tank*, Jul. 11, 2016. https://thegoldfishtank.com/buy/buy-goldfish/should-you-get-a-goldfish/

[30] "Perfect Goldfish Water Temperature: Ideal Temp Range," *The Aquarium Life*, Nov. 21, 2022. https://www.theaquariumlife.com/tips/goldfish-temperature/#:~:text=Goldfish%20thrive%20in%20freshwater%20that (accessed Feb. 22, 2024).

[31] "Making Time for Horses," *JulieGoodnight*, Dec. 02, 2016. https://signin.juliegoodnight.com/articles/free-articles/julies-blog/making-time-for-horses/ (accessed Feb. 23, 2024).

[32] "How Much Space Does a Horse Need?" *Horse Lover's Math*, May 26, 2021. https://www.horseloversmath.com/how-much-space-does-a-horse-need/

[33] "Average Salary to Comfortably Afford Horses?" *Chronicle Forums*, Apr. 19, 2019. https://forum.chronofhorse.com/t/average-salary-to-comfortably-afford-horses/457416 (accessed Feb. 22, 2024).

[34] P. Eaton, "How Do I Know If My Horse Is cold?" *Ranvet*, Jun. 22, 2021. https://www.ranvet.com.au/how-do-i-know-if-my-horse-is-cold-2/#:~:text=The%20ambient%20temperature%20that%20feels (accessed Feb. 22, 2024).

[35] "Does My Parrot Require a Daily Routine?" *Tropical Birdland*. https://tropicalbirdland.com/daily-routine/

[36] RSPCA, "How Should I House My bird?" *RSPCA Knowledgebase*, Jun. 17, 2023. https://kb.rspca.org.au/knowledge-base/how-should-i-house-my-bird/

[37] R. Colton, "Disposable Income," *The Blue Parrot*, Nov. 04, 2014. https://www.theblueparrot.info/disposable-income/ (accessed Feb. 22, 2024).

[38] S. Bell, "Parrots and the Cold Weather," *ExoticDirect*, Dec. 15, 2020. https://exoticdirect.co.uk/news/parrots-cold-weather/#:~:text=According%20to%20Dr%20Sophie%20Bell (accessed Feb. 22, 2024).

[39] "Optimum Feline Nutrition," *Cat Care Center of Baton Rouge*, Jul. 28, 2016. https://www.catcarecenter.com/services/cats/feline-nutrition-cat-food (accessed Feb. 22, 2024).

[40] Rose City Dental Care, "Who Has the Strongest Jaws in the Animal Kingdom?" *Rose City Dental Care*, Mar. 02, 2022. https://www.rosecitydentalcare.com/post/who-has-the-strongest-jaws-in-the-animal-kingdom-with-your-portland-or-family-general-dentist#:~:text=%23FunFact%3A%20Unlike%20jaguars%2C%20their (accessed Feb. 22, 2024).

[41] L. Bales and V. Higgs, "What Is the Weight of an Average Cat, and When Is My Kitty Obese?" *www.petmd.com*, Feb. 09, 2023. https://www.petmd.com/cat/general-health/average-weight-cats (accessed Feb. 22, 2024).

[42] “Estimated Energy Requirements Resting Energy Requirements Feline.” Available : https://wilmotveterinaryclinic.com/wp-content/uploads/2016/10/Resting-Energy-Requirements.pdf

[43] Blackburn Romey, “What Dog Has the Strongest Bite?” *Blackburn Romey - Trusted Injury Lawyers*, Sep. 19, 2023. https://www.blackburnromey.com/what-dog-has-the-strongest-bite/ (accessed Feb. 23, 2024).

[44] “Dog Weight Chart: Weights of Popular Dog Breeds,” *dogslim.com*. https://dogslim.com/articles/dog-weight-chart.html (accessed Feb. 22, 2024).

[45] “Goldfish Basics,” *www.americanaquariumproducts.com*. https://www.americanaquariumproducts.com/Goldfish.html (accessed Feb. 22, 2024).

[46] K. Mohr, “Goldfish, Facts and Photos,” *National Geographic*, Oct. 09, 2019. https://www.nationalgeographic.com/animals/fish/facts/goldfish (accessed Feb. 22, 2024).

[47] K. Williams, “Why Does My Horse Lack Energy?” *Dengie Horse Feeds*, Jul. 24, 2020. https://dengie.com/news-articles/feed-advice/why-does-my-horse-lack-energy/ (accessed Feb. 23, 2024).

[48] “The Modern Day Horse,” University of Guelph. Accessed: Feb. 22, 2024. [Online]. Available: https://www.equineguelph.ca/pdf/courses/trainer_kit/Modern_Day_Horse_notes.pdf

[49] “How Much Does a Horse weigh?” *www.equishop.com*, Aug. 02, 2023. https://www.equishop.com/en/blog/how-much-does-a-horse-weigh-n383#:~:text=Average%20horse%20weight%20for%20a (accessed Feb. 22, 2024).

[50] “Energy Levels Basics,” *Parrot Problem Solving 101*, 2020. https://www.parrotproblemsolving101.com/diet-basics (accessed Feb. 22, 2024).

[51] “How Bad Do Bird Bites hurt?” *ReptileKnowledge*. https://www.reptileknowledge.com/reptile-pedia/how-bad-do-bird-bites-hurt (accessed Feb. 22, 2024).

[52] “Parrot Weight - a Comprehensive Guide,” *Parrot Junkie*. https://parrotjunkie.com/blogs/health/parrot-weight (accessed Feb. 23, 2024).

[53] “Trends in the Pet Population,” *HealthforAnimals*, Jan. 28, 2022. https://healthforanimals.org/reports/pet-care-report/global-trends-in-the-pet-population/#:~:text=There%20are%20likely%20over%20a (accessed Feb. 22, 2024).

[54] M. Megna, “Pet Ownership Statistics 2024,” *www.forbes.com*, Jan. 25, 2024. https://www.forbes.com/advisor/pet-insurance/pet-ownership-statistics/ (accessed Feb. 23, 2024).

9.0 Appendices

Appendix 1 – Code for Radar Chart and Geometric Analysis

```
from shapely.geometry import Polygon
import matplotlib.pyplot as plt
import numpy as np

def values_to_polygon(values, angles):
    """Convert radar chart values and angles to a Shapely Polygon."""
    points = [[value * np.cos(angle), value * np.sin(angle)] for value, angle in zip(values, angles)]
    return Polygon(points)

def plot_radar_chart(categories, household_values, ideal_values, angles):
    """Plot a radar chart comparing household to ideal values and return polygons for area calculation."""
    fig, ax = plt.subplots(figsize=(8, 8), subplot_kw=dict(polar=True))
    plt.xticks(angles[:-1], categories, color='grey', size=12)
    ax.set_rlabel_position(0)
    plt.yticks([20, 40, 60, 80, 100], ["20", "40", "60", "80", "100"], color="grey", size=10)
    plt.ylim(0,100)

    # Convert values to Shapely polygons for area calculations
    household_polygon = values_to_polygon(household_values, angles[:-1])  # Exclude the duplicate closing angle
    ideal_polygon = values_to_polygon(ideal_values, angles[:-1])

    # Plot the household and ideal polygons on the radar chart
    household_coords = np.array(household_polygon.exterior.coords)
    ideal_coords = np.array(ideal_polygon.exterior.coords)
    ax.plot(angles, household_values + household_values[:1], color="b", linewidth=2, linestyle='solid', label='Household')
    ax.fill(angles, household_values + household_values[:1], color="b", alpha=0.4)
    ax.plot(angles, ideal_values + ideal_values[:1], color="r", linewidth=2, linestyle='solid', label='Ideal/Benchmark')
    ax.fill(angles, ideal_values + ideal_values[:1], color="r", alpha=0.4)

    plt.legend(loc='upper right', bbox_to_anchor=(1.1, 1.1))
    plt.title('Household vs. Ideal Readiness for Pet Ownership', size=15, color='black', y=1.1)
    plt.show()

    return household_polygon, ideal_polygon

def calculate_areas(household_polygon, ideal_polygon):
    """Calculate and return the areas of interest using Shapely."""
    ideal_area = ideal_polygon.area
    intersection_area = household_polygon.intersection(ideal_polygon).area
    outside_area = household_polygon.difference(ideal_polygon).area
    return ideal_area, intersection_area, outside_area

# Define categories, household and ideal values, and set up angles
categories = ['Free Time (F)', 'Household Space (H)', 'Disposable Income (Di)',
              'Animal Experience (Ac)', 'Climate (T)', 'Distance from Vet (Dv)']
household_values = [95.52, 87.83, 77.8, 59.37, 38.59, 42.06]
ideal_values = [95.22, 2.99, 38.78, 64.59, 60.09, 45.64]
N = len(categories)
angles = np.linspace(0, 2 * np.pi, N, endpoint=False).tolist()  # Full circle divided by categories
angles += angles[:1]  # Closing the loop

# Plot radar chart and calculate areas
household_polygon, ideal_polygon = plot_radar_chart(categories, household_values, ideal_values, angles)
ideal_area, intersection_area, outside_area = calculate_areas(household_polygon, ideal_polygon)

print(f"Ideal Area: {ideal_area:.2f} square units")
print(f"Intersection Area: {intersection_area:.2f} square units")
print(f"Household Outside Ideal Area: {outside_area:.2f} square units")
```

Appendix 2 – Regression used in Distribution Function Development

Figure 12 –Used for conversion from household floor area to income for accessible percentile data.

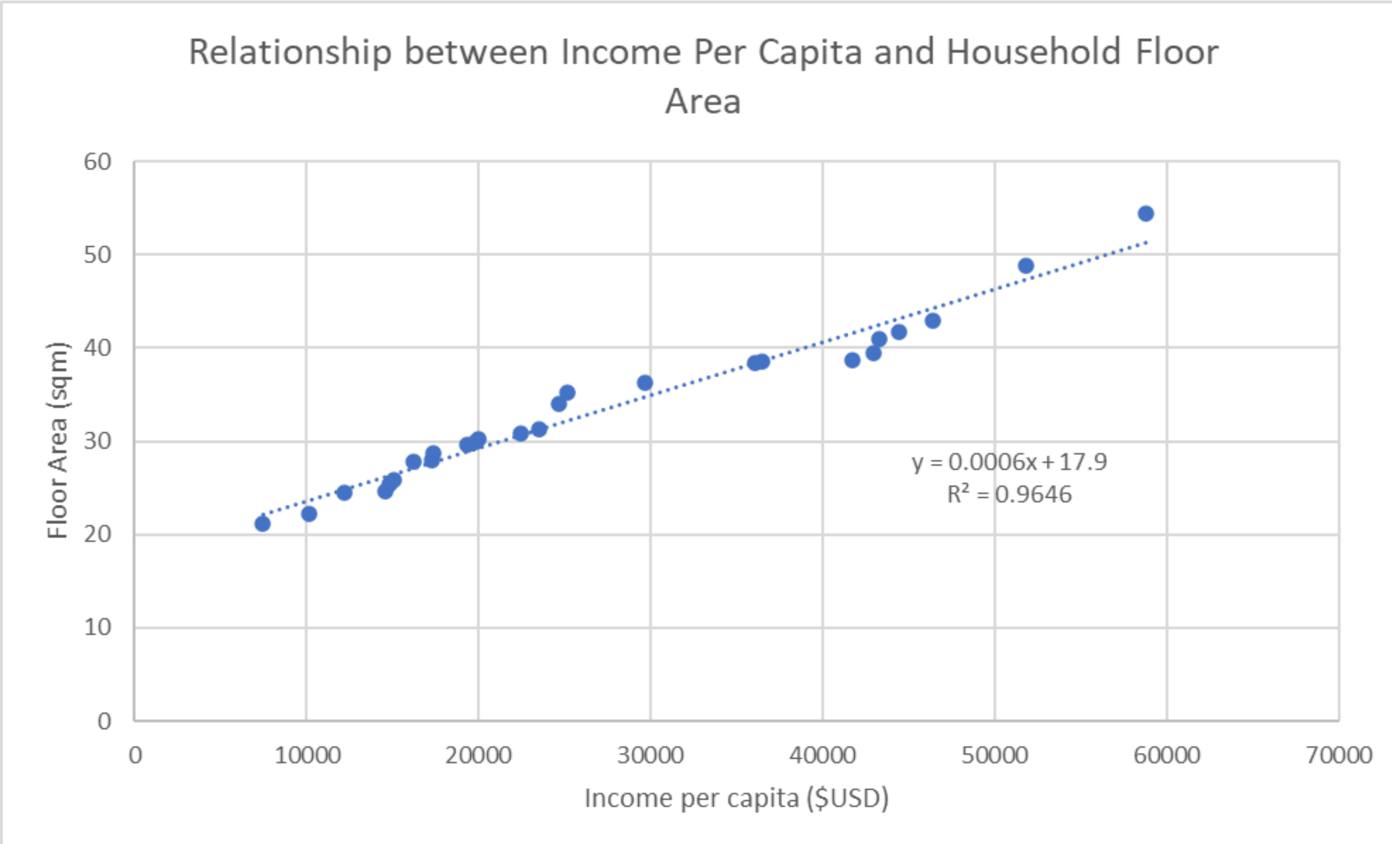


Figure 13 - Distribution function for disposable income found using regression.

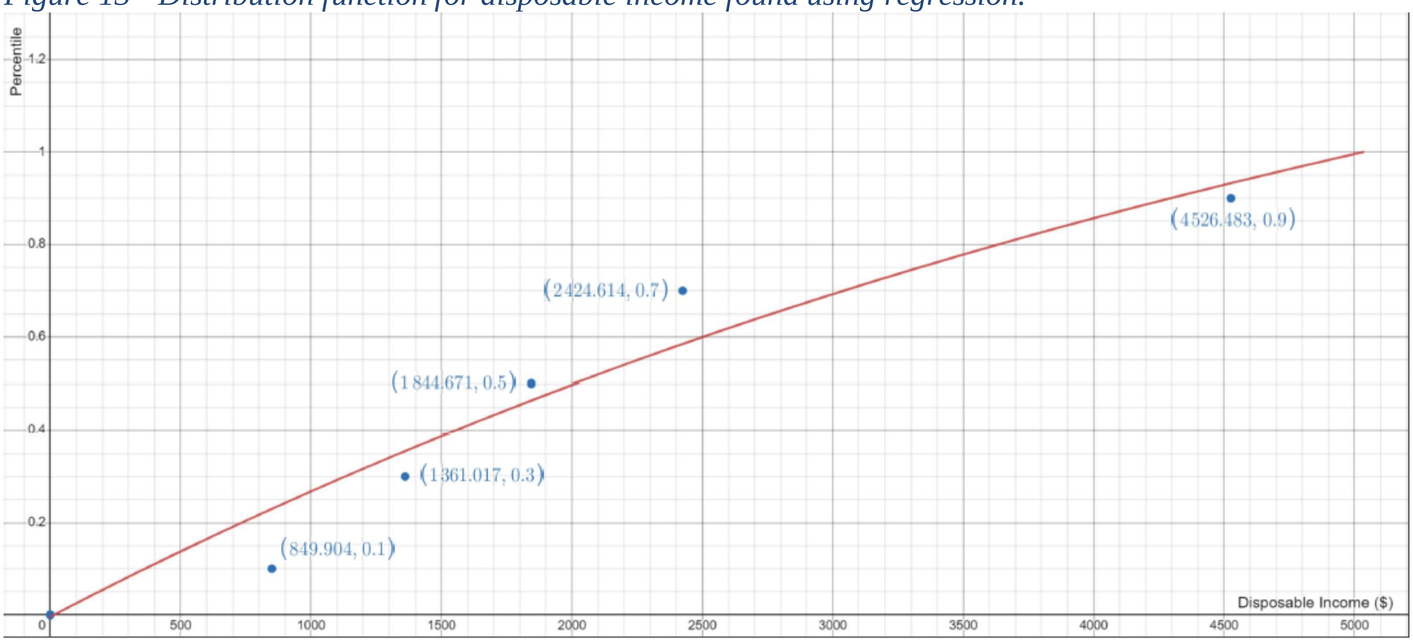
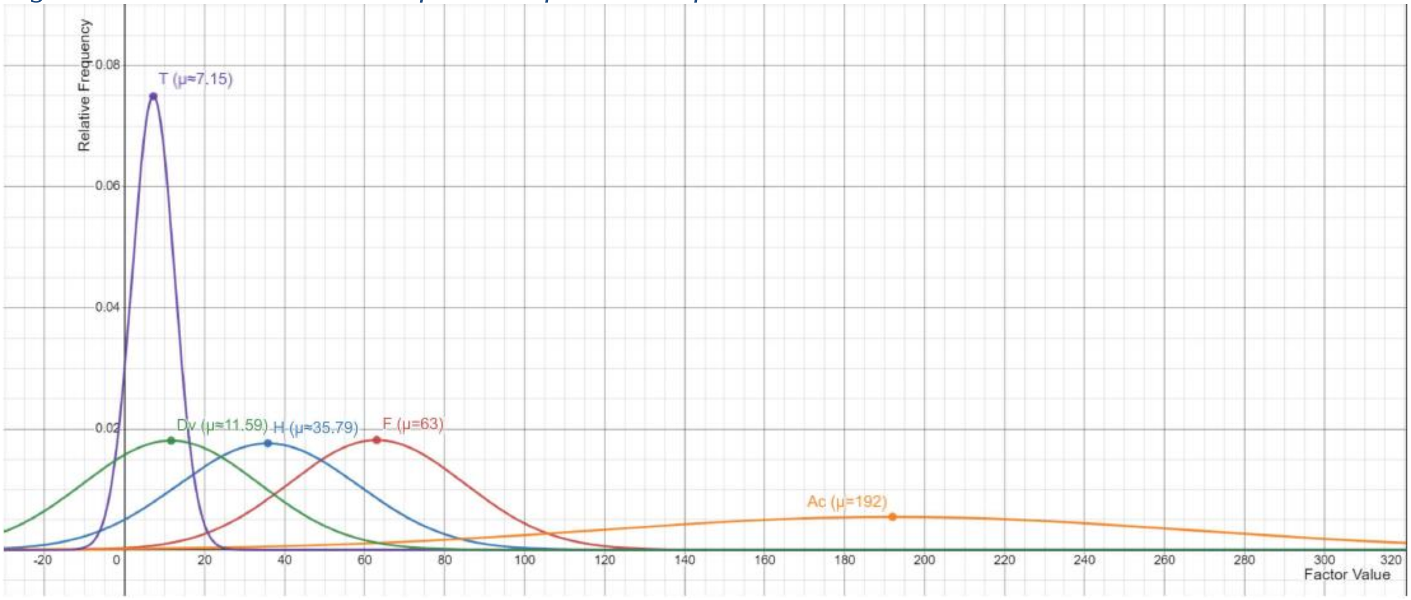


Figure 14 - Normal distribution functions for various factors.

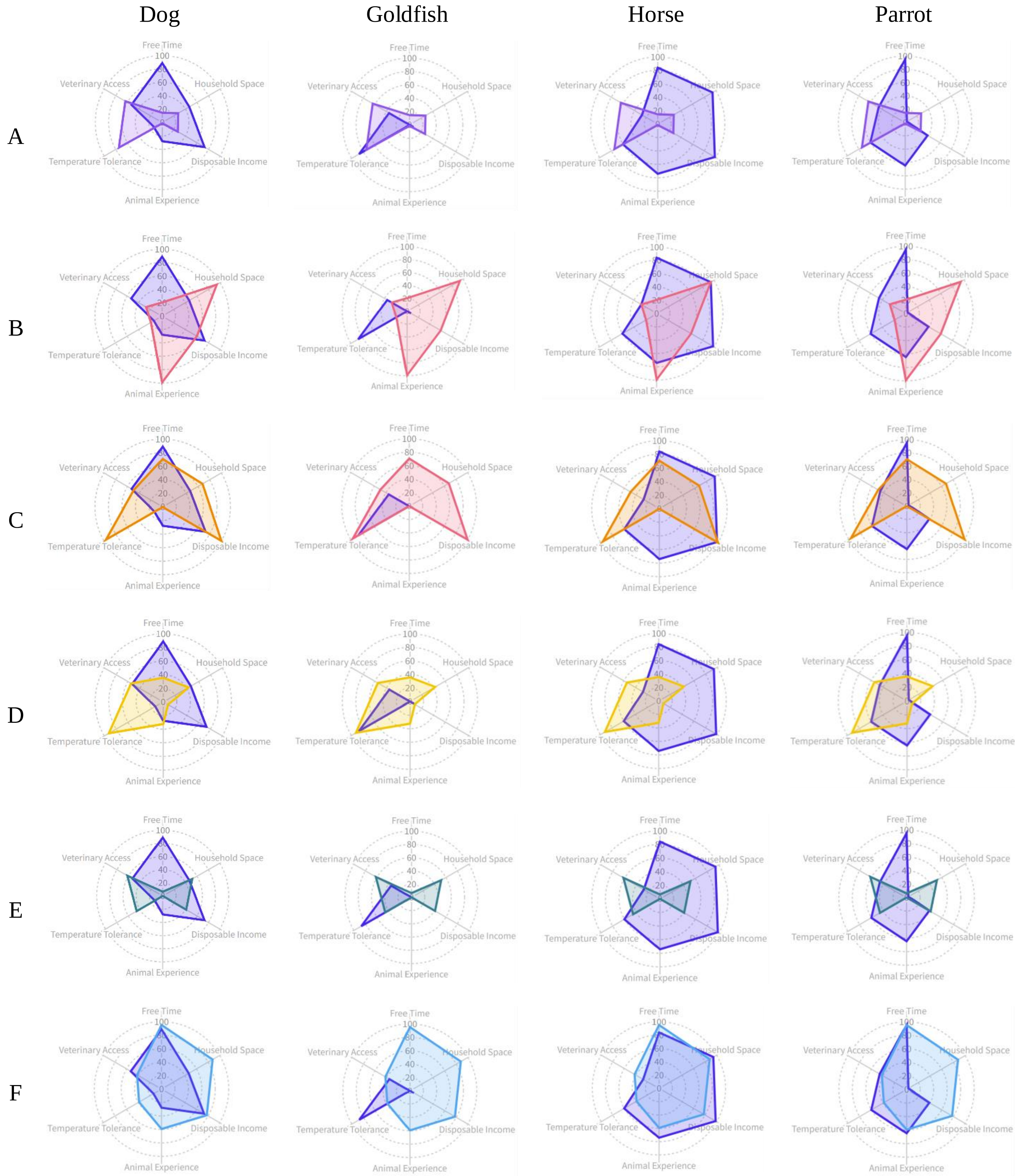


Appendix 3 – Score Calculation Code

```
import numpy as np
from shapely.geometry import Polygon
import matplotlib.pyplot as plt

def values_to_polygon(values, angles):
    """Convert radar chart values and angles to a Shapely Polygon."""
    points = [[value * np.cos(angle), value * np.sin(angle)] for value, angle in zip(values, angles)]
    return Polygon(points)

def calculate_areas_and_score(household_values, ideal_values, angles):
    """Calculate areas and score for a single household against an ideal benchmark."""
    household_polygon = values_to_polygon(household_values, angles[:-1])
    ideal_polygon = values_to_polygon(ideal_values, angles[:-1])

    # Calculate areas
    ideal_area = ideal_polygon.area
    intersection_area = household_polygon.intersection(ideal_polygon).area
    outside_area = household_polygon.difference(ideal_polygon).area

    # Calculate the score using the provided formula
    score = (intersection_area / ideal_area) + (1/5) * np.log((outside_area/ideal_area) + 1/10) + 1/5

    return ideal_area, intersection_area, outside_area, score

# Provided household values
household_values = [35.56, 42.94, 8.24, 32.57, 92.29, 54.67]

# Provided ideal values for comparison
ideal_values = [44.35, 30.63, 73.01, 53.97, 27.08, 52.87]

# Categories and angles for the radar chart
categories = ['Free Time (F)', 'Household Space (H)', 'Disposable Income (Di)',
              'Animal Experience (Ac)', 'Temperature Tolerance (T)', 'Distance from Vet (Dv)']
N = len(categories)
angles = np.linspace(0, 2 * np.pi, N, endpoint=False).tolist()
angles += angles[:1]  # Closing the loop

# Calculate areas and score for the household
ideal_area, intersection_area, outside_area, score = calculate_areas_and_score(household_values, ideal_values, angles)

# Printing the results
print(f"Ideal Area: {ideal_area:.2f} square units")
print(f"Intersection Area: {intersection_area:.2f} square units")
print(f"Outside Ideal Area: {outside_area:.2f} square units")
print(f"Score: {score:.2f}")
```

Appendix 4 – Radar Charts of Selected Animals Against Households

Figure 14 - Compilation of Radar Charts Generated Throughout Compatibility Analysis



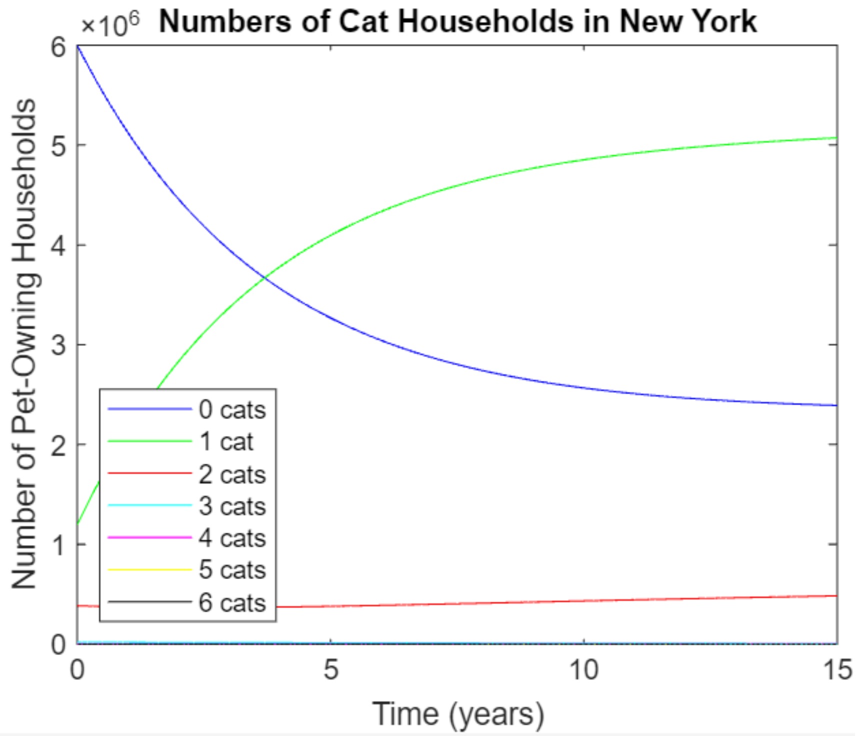
Appendix 5 – MATLAB Code for Population Projection

```
format shortE
Lc = 0.083;
Ld = 0.079;
Lg = 0.2;
Lh = 0.057;
Lp = 0.019;
CCNY = 38743234;
GRNY = 0.008;
CCS = 36289840;
GRS = 0.018;
CCC = 8885896;
GRC = 0.007;
W = [0.586343197 0.091689263 0.014337884 0.002242083 0.000350605 0.0000548258];
%Rc
RNYSc = [0.323925851 0.104078805 0.005379775 0.000431537 0.000142768 0.000110036];
RCc = [0.102492375 0.013517595 0.000134953 0.00000291087 0.000000221237 0.00000014873];
%Rd
RNYSd = [0.238638515 0.048884755 0.002427158 0.000910107 0.000244056
0.0000857548426487];
RCd = [0.056186567 0.002256687 0.00000666062420011 0.00000183217512979 0.000000451069
0.000000126527];
%Rg
RNYSg = [0.273865237 0.286512833 0.300528516 0.315219741 0.330221659 0.3453052];
RCg = [0.712768115 0.693711246 0.672439623 0.649925132 0.626641422 0.602846875];
%Rh
RNYSh = [0.058353511354 0.0000185041099684 0.000262406322981 0.000115050123822
0.000143470800573 0.000167307594933];
RCh = [0.0025566098666 0.0000272430670603 0.00000096073 0.0000015707 0.000000191799
0.000000223567];
%Rp
RNYSp = [0.326814444 0.100219649 0.012231433 0.001534591 0.000836435 0.000466943];
RCp = [0.081975798 0.002007288 0.0000245623551936 0.00000227561802975
0.00000113408337372 0.000000629819];
%t0c
t0NYc = [5999969 1197406.194 384731.8907 19886.57416 1595.194665 527.7477215
406.7521519];
t0Sc = [1274896 93061.21214 29900.97787 1545.56466 123.9769344 41.01602525
31.61237055];
t0Cc = [295408 42435.69371 5596.792166 55.8757326 1.205210569 0.091600499 0.061579963];
%t0d
t0NYd = [5551301.79 1682671.375 344692.8009 17114.20891 6417.283358 1720.873503
604.668609];
t0Sd = [1159428.64 196827.0494 40319.7368 2001.899654 750.6486219 201.2956653
70.72987629];
t0Cd = [274798.40 66036.63244 2652.307033 7.828298057 2.153373705 0.530145428
0.148708883];
%t0g
t0NYg = [6083618.40 224946.4974 235334.9371 246847.0911 258914.1192 271236.344
283625.6112];
t0Sg = [1007712.00 57961.44806 60638.21348 63604.5238 66713.80727 69888.84666
73081.16073];
```

```
t0Cg = [230143.66 20411.46391 19865.73441 19256.58124 18611.83617 17945.06307
17263.66119];
%t0h
t0NYh = [7404523.00 197606.7187 62.66180679 888.6055227 389.6025608 485.8455554
566.5658173];
t0Sh = [1397600.00 1976.067176 0.626618064 8.886055175 3.896025585 4.858455526
5.66565814];
t0Ch = [340993.75 2476.372163 26.38805935 0.930578776 0.15214548 0.185779208
0.216550841];
%t0p
t0NYp = [7148251.62 337287.7141 103431.3411 12623.40828 1583.768948 863.2402653
481.9073619];
t0Sp = [1357612.00 31038.62561 9518.184438 1161.658807 145.7450401 79.43897831
44.34713024];
t0Cp = [329758.08 13406.95483 328.2873999 4.017117168 0.372172138 0.185476749
0.103005496];
%ODEs
dPdt = @(t,P) [ GRC*P(1)*(1 - P(1)/CCC) + Lp*P(2) - RCp(1)*W(1)*P(1)
   RCp(1)*W(1)*P(1) + Lp*P(3) - Lp*P(2) - RCp(2)*W(2)*P(2)
   RCp(2)*W(2)*P(2) + Lp*P(4) - Lp*P(3) - RCp(3)*W(3)*P(3)
   RCp(3)*W(3)*P(3) + Lp*P(5) - Lp*P(4) - RCp(4)*W(4)*P(4)
   RCp(4)*W(4)*P(4) + Lp*P(6) - Lp*P(5) - RCp(5)*W(5)*P(5)
   RCp(5)*W(5)*P(5) + Lp*P(7) - Lp*P(6) - RCp(6)*W(6)*P(6)
   RCp(6)*W(6)*P(6) - Lp*P(7)];
tinterval = 0:0.1:15;
[t, P] = ode45(dPdt, tinterval, t0Cp);
clf
plot(t,P(:,1),'b',t,P(:,2),'g',t,P(:,3),'r',t,P(:,4),'c',t,P(:,5),'m',t,P(:,6),'y',t,P(
:,7),'k')
legend({'0 parrots','1 parrot','2 parrots','3 parrots','4 parrots','5 parrots','6
parrots'},'Location','southwest')
title('Numbers of Parrot Households in Copenhagen')
xlabel('Time (years)')
ylabel('Number of Pet-Owning Households')
%Get 51st, 101st, and 151st rows from P
```

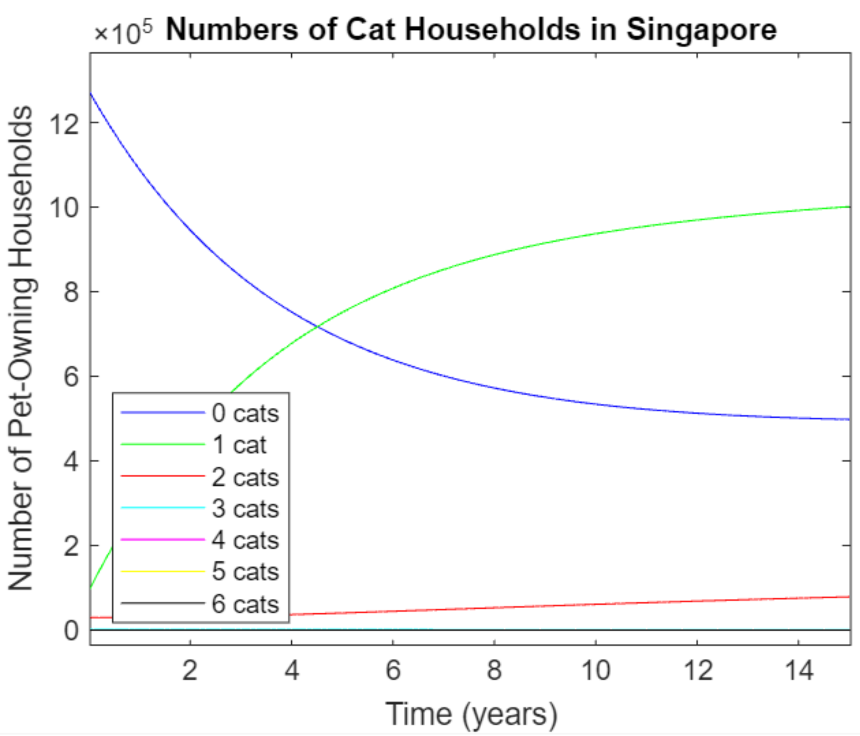
Appendix 6 – Population Projection Results for Various Pets and Regions

New York Cats



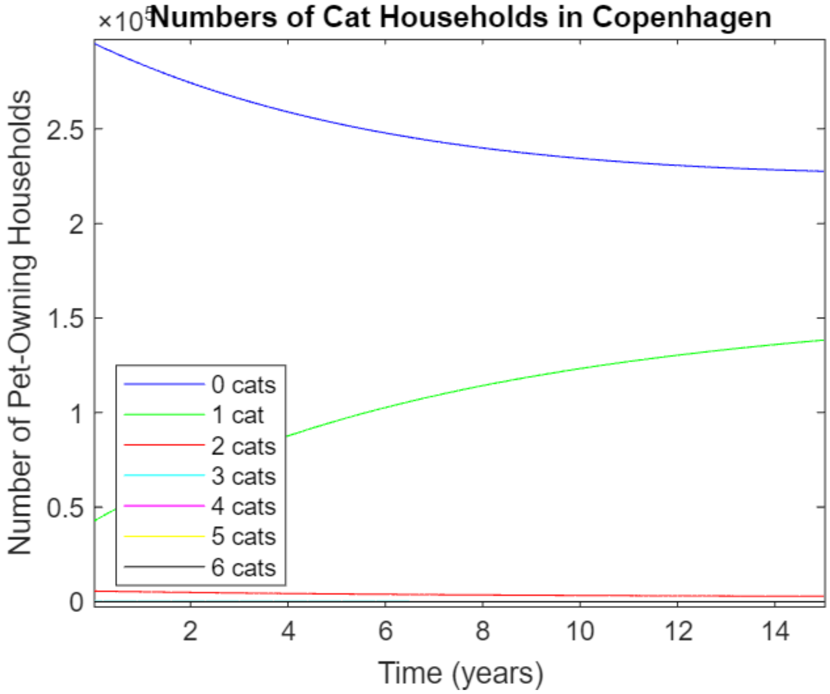
Years	5	10	15
0 cats	3268018	2566426	2391471
1 cat	4095143	4852749	5072569
2 cats	379029	432855	483256
3 cats	13719	9550	6734
4 cats	1221	947	739
5 cats	459	377	297
6 cats	268	177	117

Singapore Cats



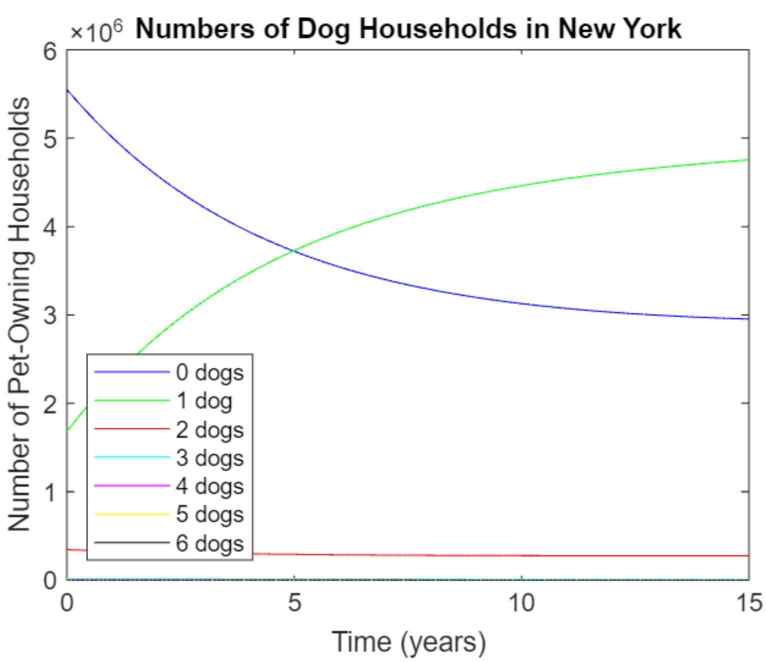
Years	5	10	15
0 cats	687558	533997	497887
1 cat	751066	937007	1001058
2 cats	40215	60750	78413
3 cats	1067	749	539
4 cats	94	73	57
5 cats	35	29	23
6 cats	20	13	9

Copenhagen Cats



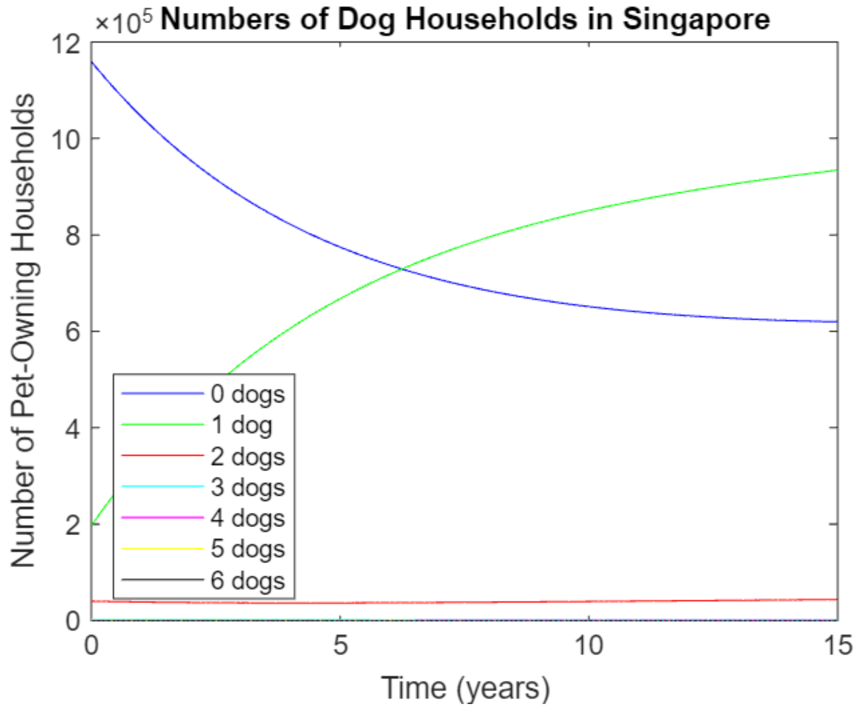
Years	5	10	15
0 cats	252941	234402	227616
1 cat	95640	123252	138336
2 cats	4085	3275	2839
3 cats	37	24	16
4 cats	0	0	0
5 cats	0	0	0
6 cats	0	0	0

New York Dogs



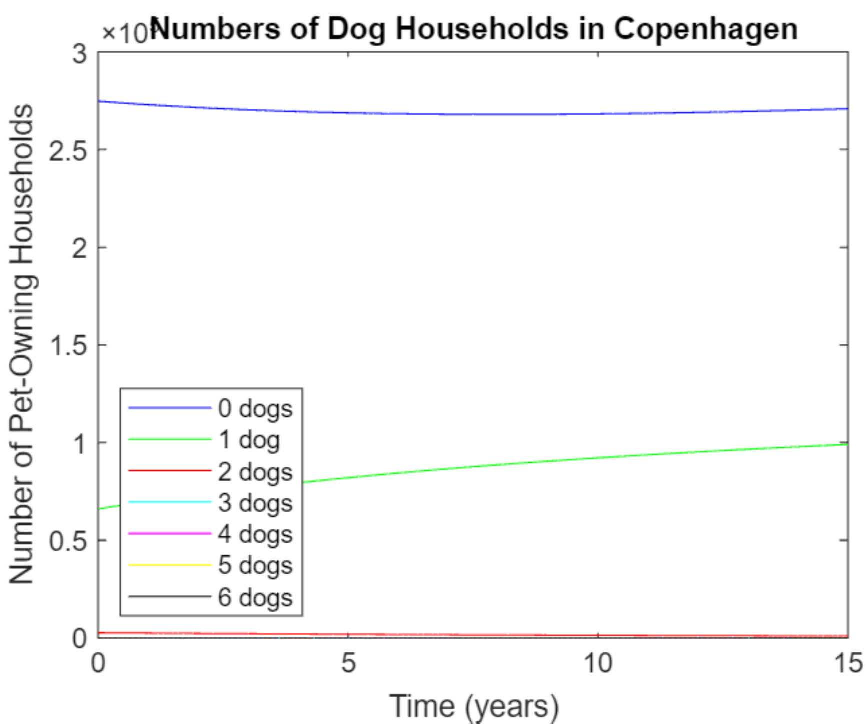
Years	5	10	15
0 dogs	3720672	3128616	2954136
1 dog	3730151	4463833	4755148
2 dogs	291663	277663	275875
3 dogs	13376	10404	8065
4 dogs	4813	3615	2715
5 dogs	1320	997	745
6 dogs	407	274	184

Singapore Dogs



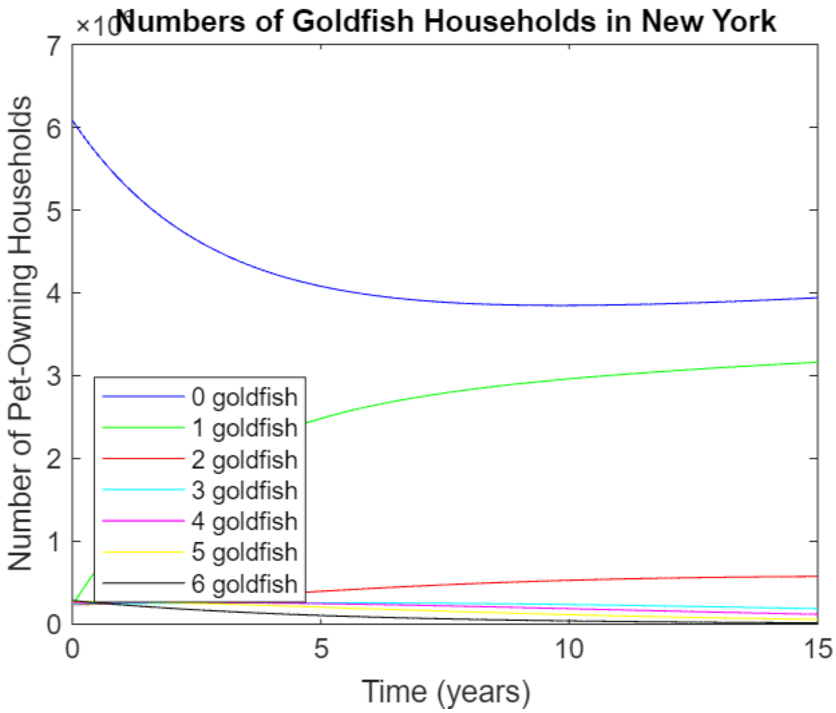
Years	5	10	15
0 dogs	774626	651237	619740
1 dog	667657	850572	934227
2 dogs	36721	39589	43668
3 dogs	1564	1217	945
4 dogs	5629	422	317
5 dogs	1544	116	87
6 dogs	476	32	21

Copenhagen Dogs



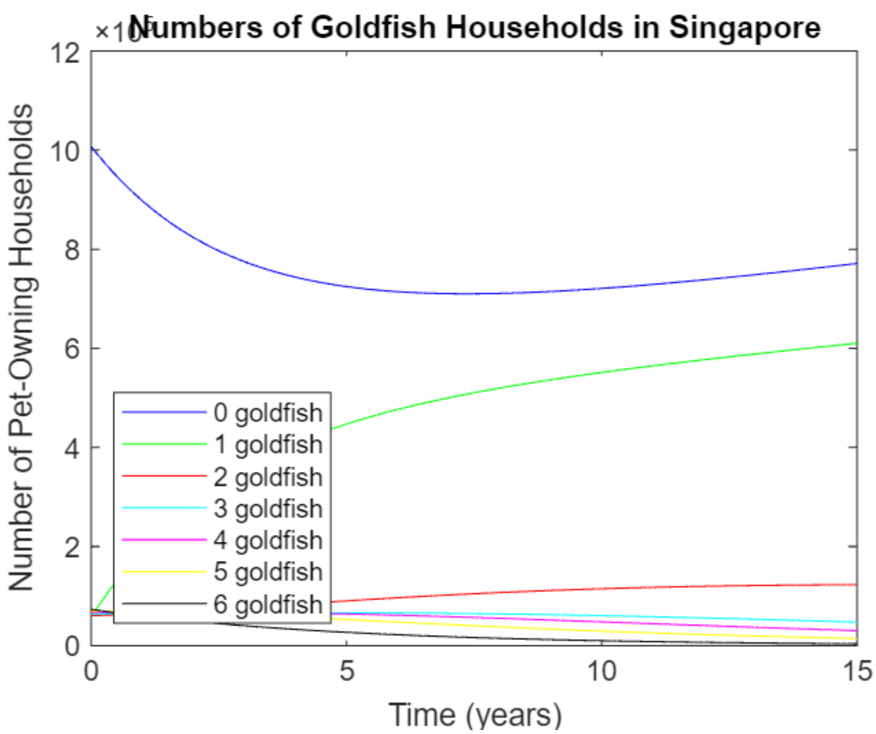
Years	5	10	15
0 dogs	268816	268287	270905
1 dog	82022	92186	99062
2 dogs	1853	1325	976
3 dogs	5	4	3
4 dogs	1	1	0
5 dogs	0	0	0
6 dogs	0	0	0

New York Goldfish



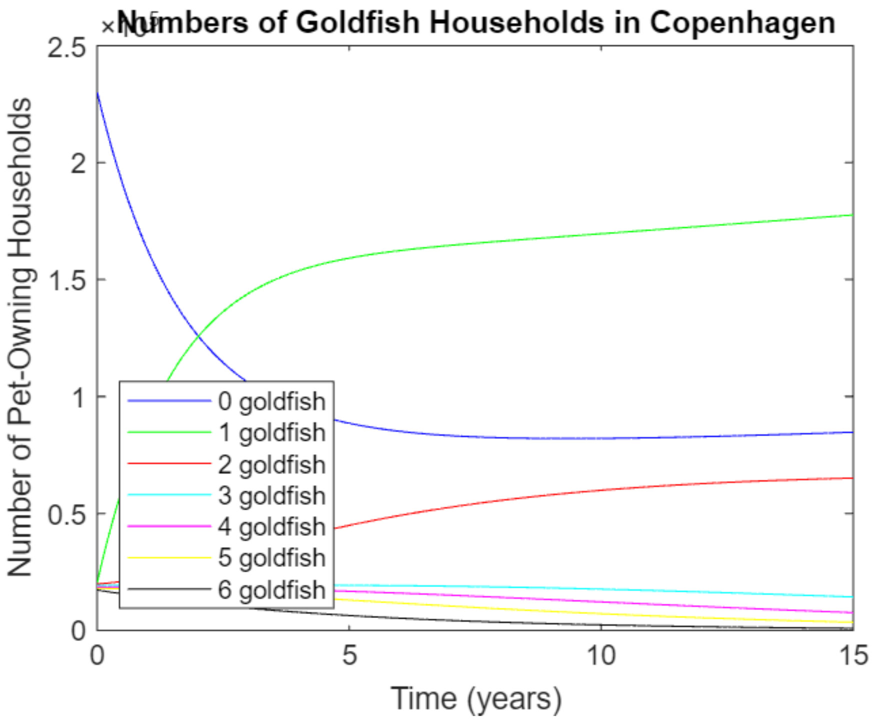
Years	5	10	15
0 fish	4080066	3848509	3940692
1 fish	2482431	2961328	3163228
2 fish	395742	528778	574774
3 fish	257333	235965	185828
4 fish	247711	185954	117692
5 fish	204208	113587	55963
6 fish	104354	38398	14130

Singapore Goldfish



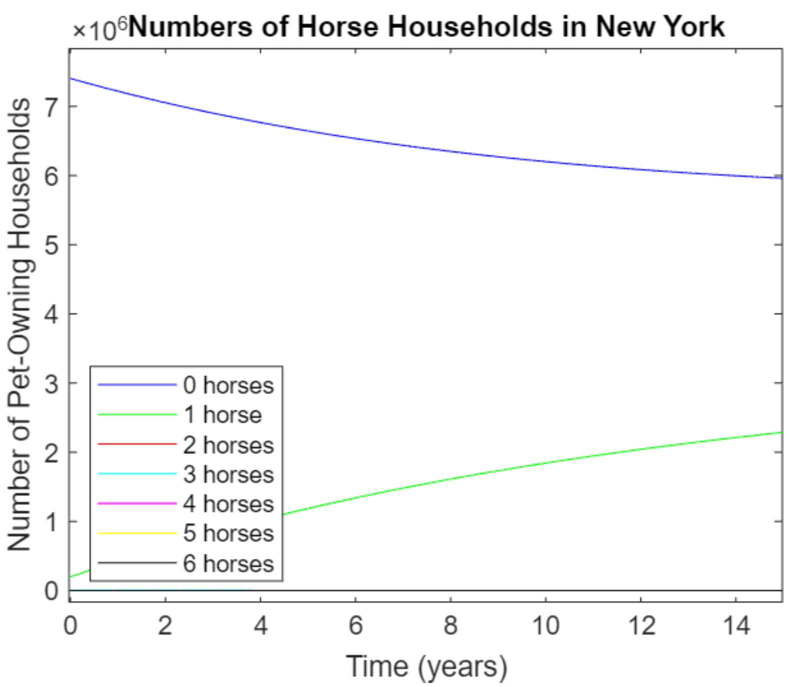
Years	5	10	15
0 fish	724646	720973	771231
1 fish	447165	551299	610223
2 fish	90290	114808	122996
3 fish	66225	60526	47455
4 fish	63827	47914	30324
5 fish	52617	29267	14419
6 fish	26888	9894	3641

Copenhagen Goldfish



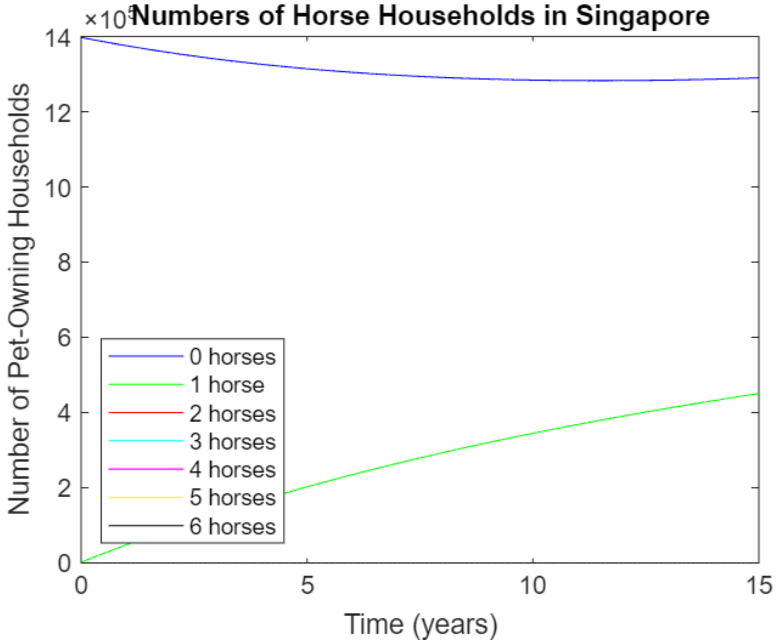
Years	5	10	15
0 fish	88566	82082	84721
1 fish	159172	169588	177639
2 fish	44891	59903	65103
3 fish	19265	17623	14299
4 fish	16705	12162	7590
5 fish	12964	7115	3484
6 fish	6352	2337	860

New York Horses



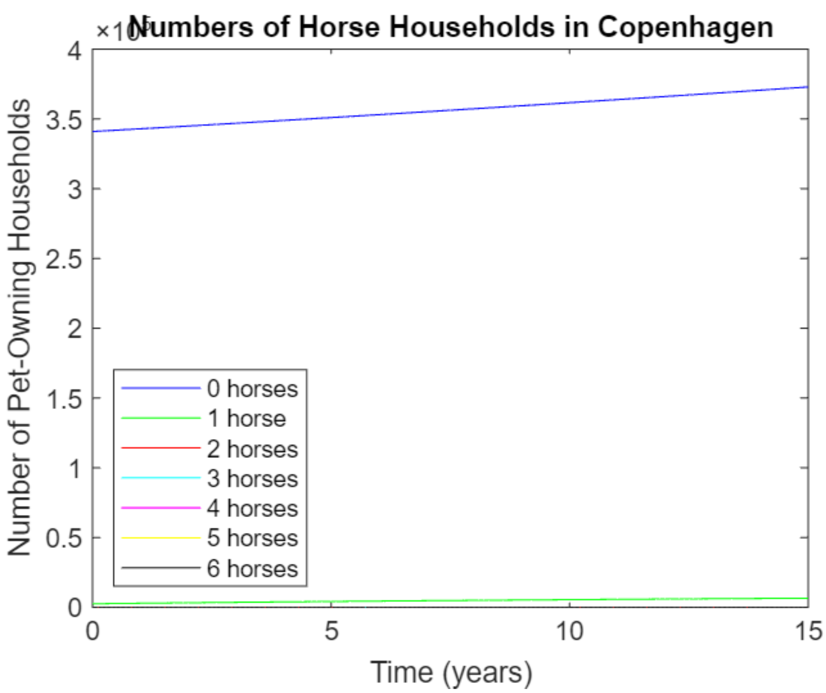
Years	5	10	15
0 horses	6644550	6201180	5960797
1 horse	1186775	1843972	2289147
2 horses	256	383	464
3 horses	768	682	620
4 horses	414	428	430
5 horses	486	457	412
6 horses	426	320	240

Singapore Horses



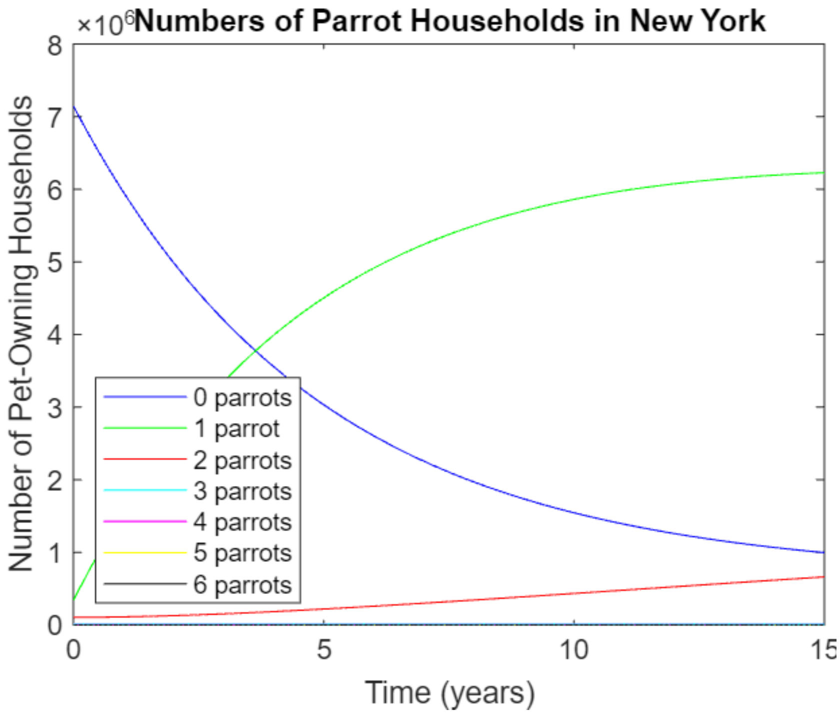
Years	5	10	15
0 horses	1341361	1284195	1289990
1 horse	202245	344858	450585
2 horses	3	6	9
3 horses	7	6	6
4 horses	4	4	4
5 horses	4	4	4
6 horses	4	3	2

Copenhagen Horses



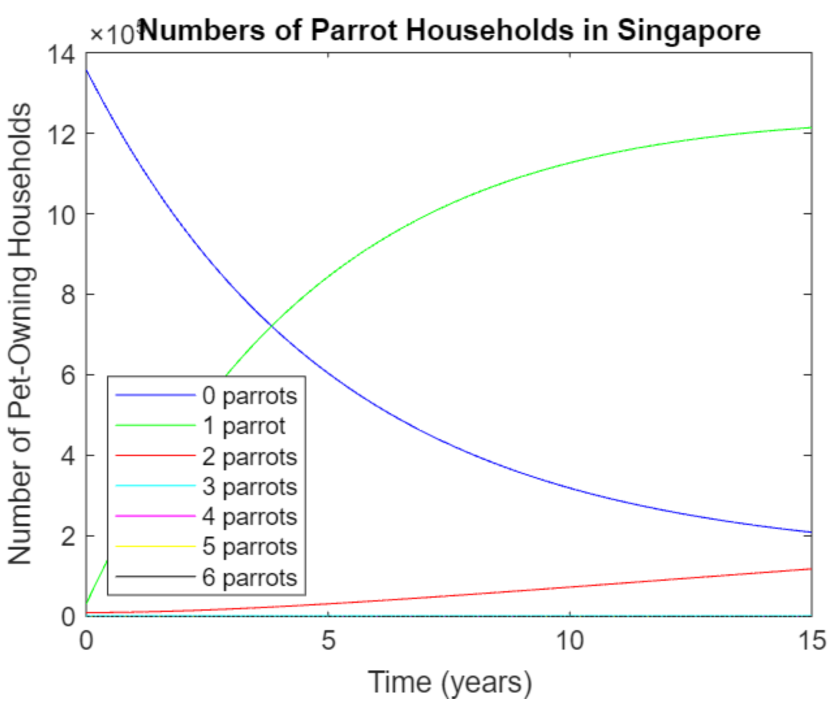
Years	5	10	15
0 horses	350987	361655	372928
1 horse	4125	5431	6484
2 horses	20	15	11
3 horses	0	0	0
4 horses	0	0	0
5 horses	0	0	0
6 horses	0	0	0

New York Parrots



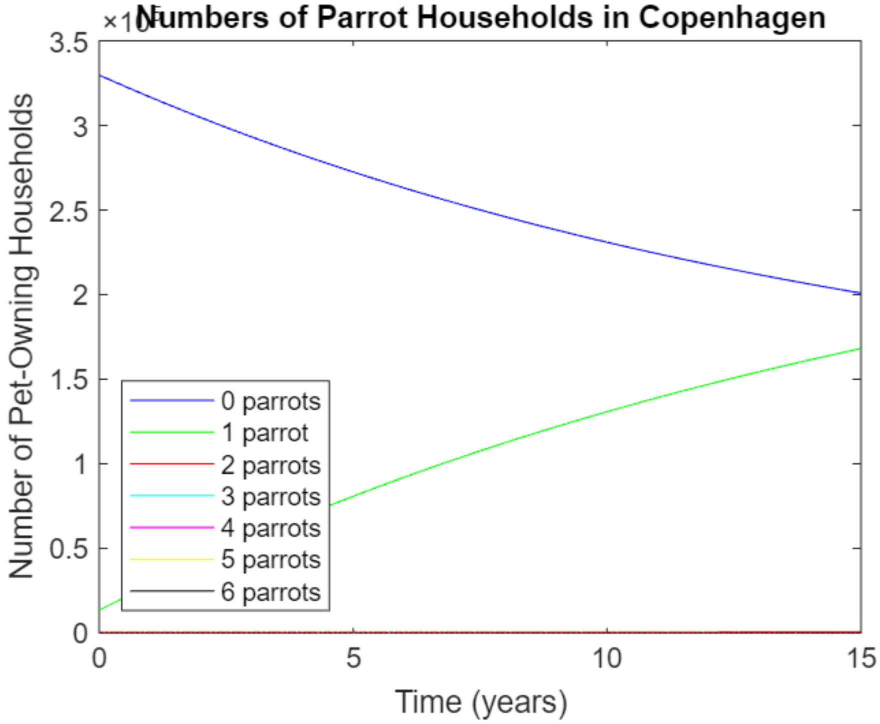
Years	5	10	15
0 Par.	3032209	1544065	995711
1 Par.	4504136	5860392	6228423
2 Par.	218774	433038	661363
3 Par.	11742	11084	10667
4 Par.	1516	1452	1391
5 Par.	826	789	752
6 Par.	438	398	362

Singapore Parrots



Years	5	10	15
0 Par.	604203	318703	208944
1 Par.	843512	1126463	1213909
2 Par.	31184	72791	117934
3 Par.	1084	1041	1038
4 Par.	139	133	128
5 Par.	76	72	69
6 Par.	40	36	33

Copenhagen Parrots



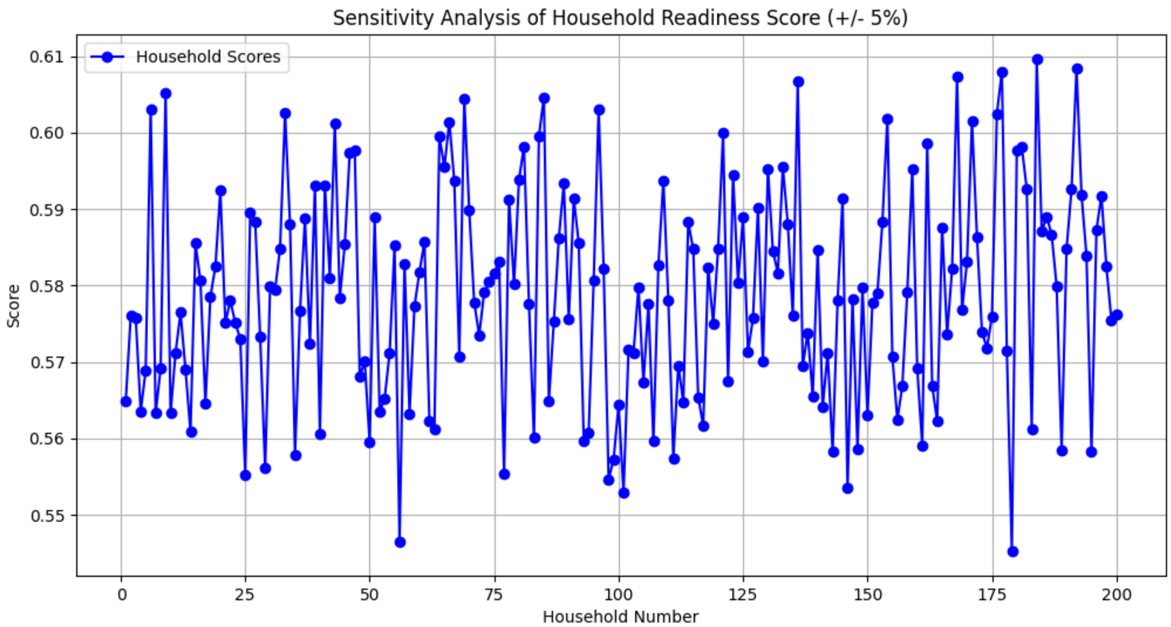
Years	5	10	15
0 Par.	272516	230971	200899
1 Par.	80766	130773	168096
2 Par.	342	405	501
3 Par.	3	3	3
4 Par.	0	0	0
5 Par.	0	0	0
6 Par.	0	0	0

Appendix 7 – Mass Household Score Simulation Code

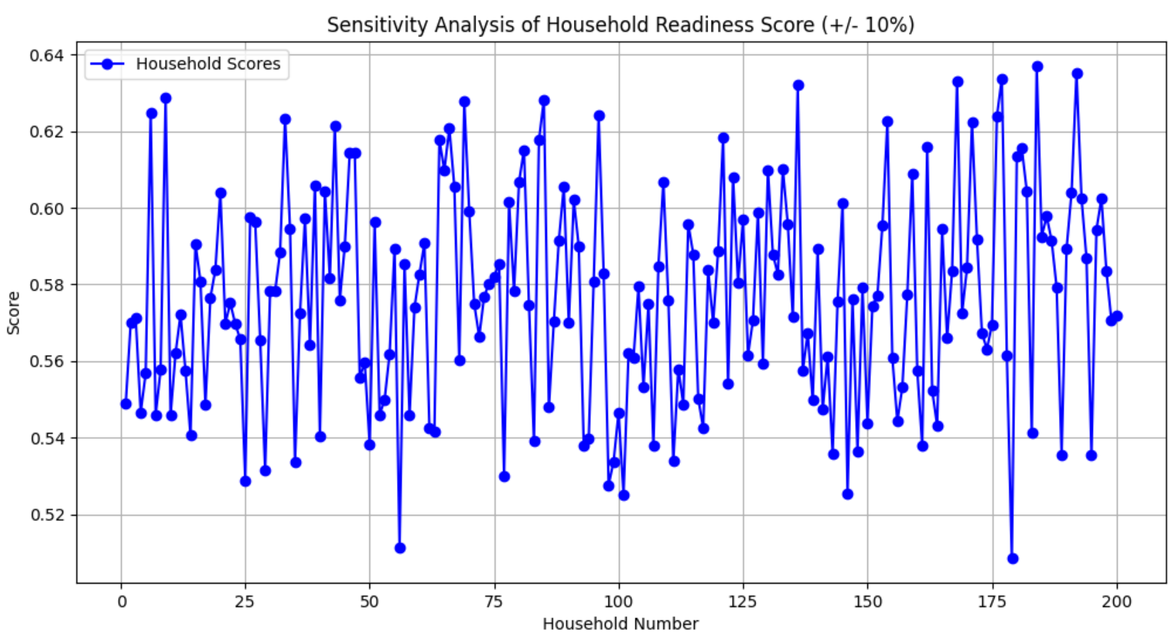
```python
import numpy as np
from shapely.geometry import Polygon
import matplotlib.pyplot as plt

def values_to_polygon(values, angles):
    """Convert radar chart values and angles to a Shapely Polygon."""
    points = [[value * np.cos(angle), value * np.sin(angle)] for value, angle in zip(values, angles)]
    return Polygon(points)

def calculate_areas_for_households(household_values_list, ideal_values, angles):
    """Calculate areas and scores for multiple households against an ideal benchmark."""
    ideal_polygon = values_to_polygon(ideal_values, angles[:-1])
    ideal_area = ideal_polygon.area

    results = []
    for household_values in household_values_list:
        household_polygon = values_to_polygon(household_values, angles[:-1])
        intersection_area = household_polygon.intersection(ideal_polygon).area
        outside_area = household_polygon.difference(ideal_polygon).area

        # Calculate the score for each household using the provided formula
        score = (intersection_area / ideal_area) + (1/5) * np.log((outside_area / ideal_area) + 1/10) + 1/5
        results.append((intersection_area, outside_area, score))

    return ideal_area, results

# Base values for generating random household values within a specified range
base_values = np.array([35.56, 42.94, 8.24, 32.57, 92.29, 54.67])
# Ideal values for comparison
ideal_values = [44.35, 30.63, 73.01, 53.97, 27.08, 52.87]

np.random.seed(42)  # Ensure reproducible random results
num_households = 200

# Generate household values list with random values within +/- 10% of the base values
household_values_list = [[np.random.uniform(low=value*0.9, high=value*1.1) for value in base_values] for _ in range(num_households)]

# Categories and angles for the radar chart
categories = ['Free Time (F)', 'Household Space (H)', 'Disposable Income (Di)',
              'Animal Experience (Ac)', 'Temperature Tolerance (T)', 'Distance from Vet (Dv)']
N = len(categories)
angles = np.linspace(0, 2 * np.pi, N, endpoint=False).tolist()
angles += angles[:1]  # Closing the loop

# Calculate areas and scores for all households
ideal_area, results = calculate_areas_for_households(household_values_list, ideal_values, angles)

# Plotting the scores for visualization
scores = [result[2] for result in results]
plt.figure(figsize=(12, 6))
plt.plot(range(1, num_households + 1), scores, marker='o', linestyle='-', color='blue', label='Household Scores')
plt.title('Sensitivity Analysis of Household Readiness Score (+/- 10%)')
plt.xlabel('Household Number')
plt.ylabel('Score')
plt.grid(True)
plt.legend()
plt.show()

# Printing detailed results for the first 5 households
print(f"Ideal Area: {ideal_area:.2f} square units")
for i, (intersection_area, outside_area, score) in enumerate(results[:200], 1):
    print(f"\nHousehold {i}:")
    print(f"  - Intersection Area: {intersection_area:.2f} square units")
    print(f"  - Outside Ideal Area: {outside_area:.2f} square units")
    print(f"  - Score: {score:.2f}")
```

Appendix 8 - Household Input Error Simulation Results

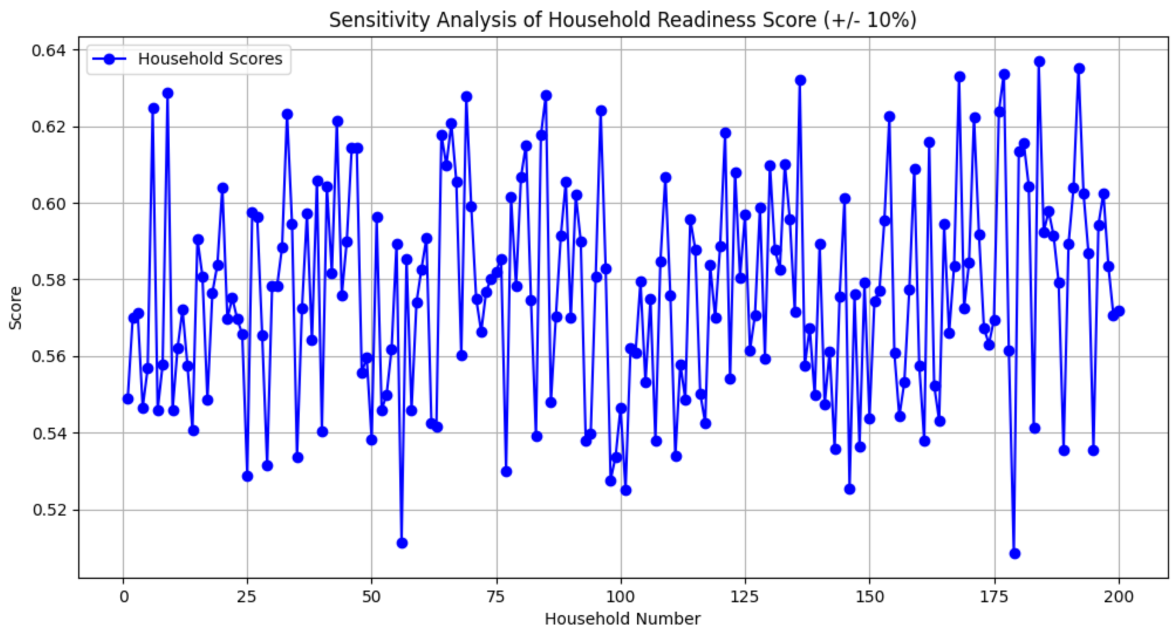
E = 0.05



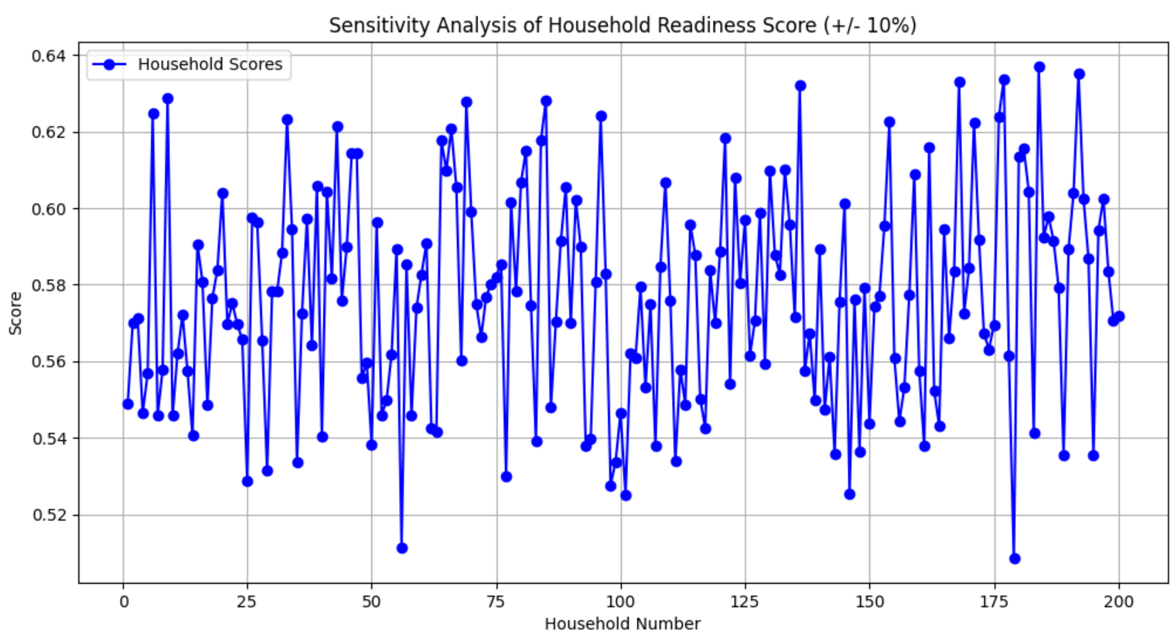
E = 0.1



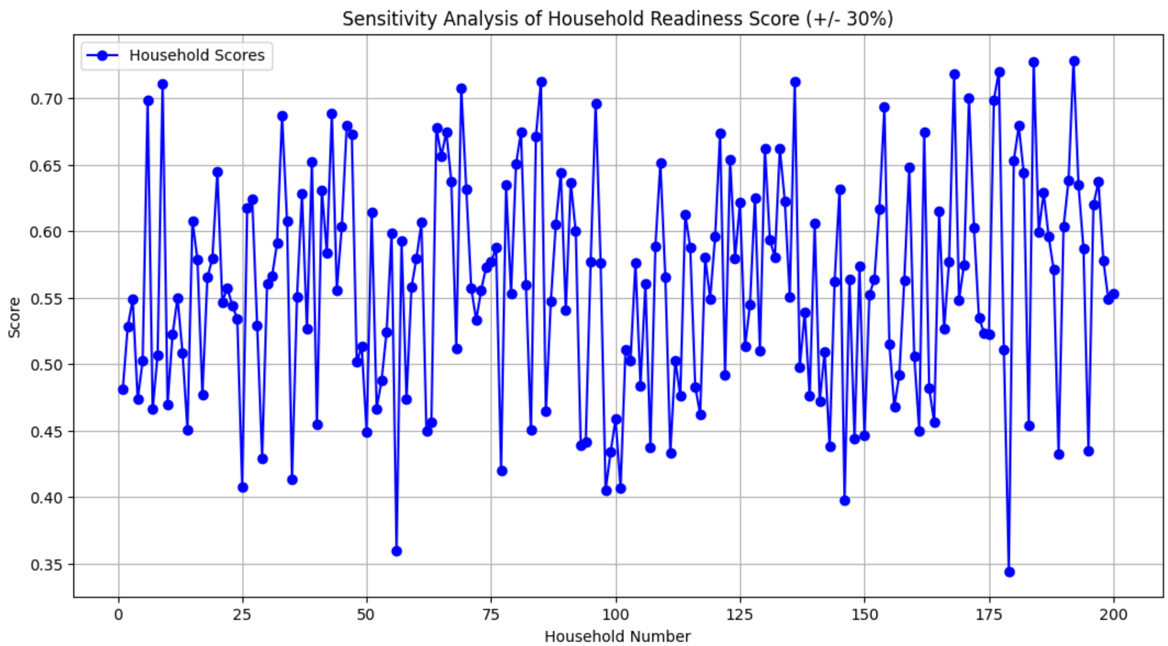
E = 0.15


Sensitivity Analysis of Household Readiness Score (+/- 10%)
Household Scores
Score
Household Number
0.64
0.62
0.60
0.58
0.56
0.54
0.52
0
25
50
75
100
125
150
175
200

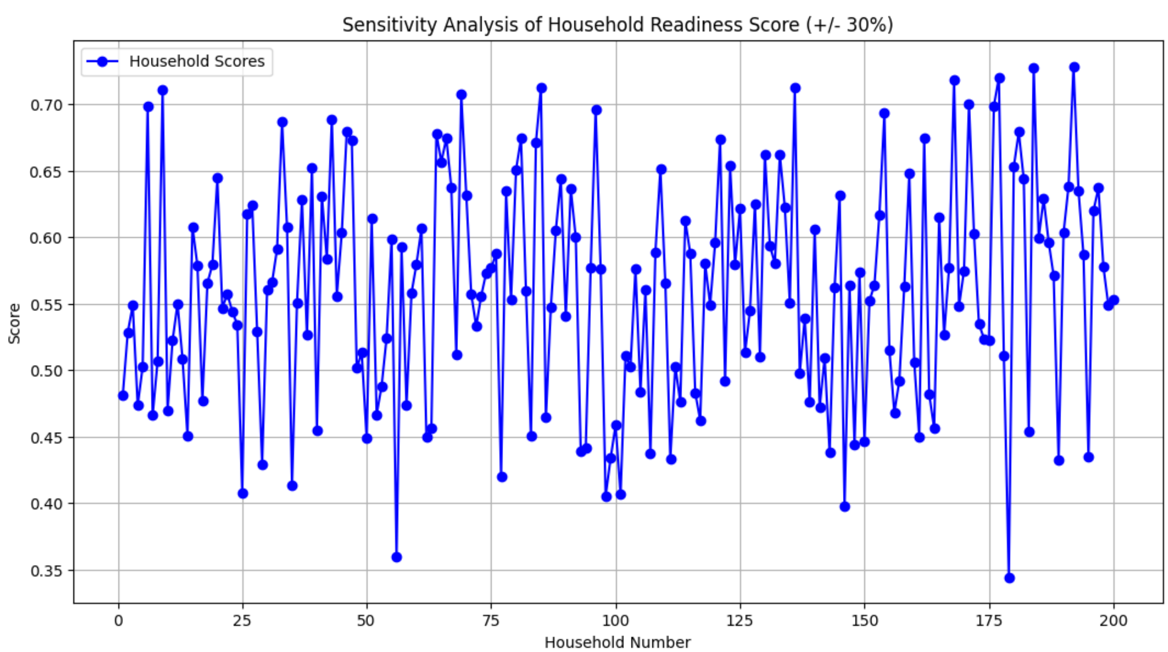
E = 0.2


Sensitivity Analysis of Household Readiness Score (+/- 10%)
Household Scores
Score
Household Number
0.64
0.62
0.60
0.58
0.56
0.54
0.52
0
25
50
75
100
125
150
175
200

 Radar **E = 0.25**



E = 0.3



Report on Use of AI

OpenAI ChatGPT (Feb 21, 2024, ChatGPT-4)
Query: Develop a code to output two radar charts for household and ideal values which also calculates the shared area, area of the ideal chart and the area of the household outside the ideal area.
Output:

ChatGPT

Given the environment's constraints and the fact that Shapely cannot be directly used here, I'll provide a complete hypothetical Python code snippet that combines everything: generating a radar chart with Matplotlib, using Shapely for advanced geometric operations like intersections, and calculating areas. This code is meant for use in an environment where you can install and run Shapely.

python Copy code

```python
# Install Shapely in your local environment: pip install shapely
from shapely.geometry import Polygon
import matplotlib.pyplot as plt
import numpy as np

def values_to_polygon(values, angles):
    """Convert radar chart values and angles to a Shapely Polygon."""
    points = [[value * np.cos(angle), value * np.sin(angle)] for value, angle in zip(
    return Polygon(points)

def plot_radar_chart(categories, household_values, ideal_values, angles):
    """Plot a radar chart comparing household to ideal values and return polygons for
    fig, ax = plt.subplots(figsize=(8, 8), subplot_kw=dict(polar=True))
    plt.xticks(angles[:-1], categories, color='grey', size=12)
    ax.set_rlabel_position(0)
    plt.yticks([20, 40, 60, 80, 100], ["20", "40", "60", "80", "100"], color="grey",
    plt.ylim(0,100)

    # Convert values to Shapely polygons for area calculations
    household_polygon = values_to_polygon(household_values, angles[:-1])  # Exclude t
    ideal_polygon = values_to_polygon(ideal_values, angles[:-1])

    # Plot the household and ideal polygons on the radar chart
```

```
    # Convert values to Shapely polygons for area calculations
    household_polygon = values_to_polygon(household_values, angles[:-1])  # Exclude t
    ideal_polygon = values_to_polygon(ideal_values, angles[:-1])

    # Plot the household and ideal polygons on the radar chart
    household_coords = np.array(household_polygon.exterior.coords)
    ideal_coords = np.array(ideal_polygon.exterior.coords)
    ax.plot(angles, household_values + household_values[:1], color="b", linewidth=2,
    ax.fill(angles, household_values + household_values[:1], color="b", alpha=0.4)
    ax.plot(angles, ideal_values + ideal_values[:1], color="r", linewidth=2, linestyl
    ax.fill(angles, ideal_values + ideal_values[:1], color="r", alpha=0.4)

    plt.legend(loc='upper right', bbox_to_anchor=(1.1, 1.1))
    plt.title('Household vs. Ideal Readiness for Pet Ownership', size=15, color='blac
    plt.show()

    return household_polygon, ideal_polygon

def calculate_areas(household_polygon, ideal_polygon):
    """Calculate and return the areas of interest using Shapely."""
    ideal_area = ideal_polygon.area
    intersection_area = household_polygon.intersection(ideal_polygon).area
    outside_area = household_polygon.difference(ideal_polygon).area
    return ideal_area, intersection_area, outside_area

# Define categories, household and ideal values, and set up angles
categories = ['Free Time (F)', 'Household Space (H)', 'Disposable Income (Di)',
              'Animal Experience (Ac)', 'Climate (T)', 'Health/Satisfaction Convenien
household_values = [75, 65, 80, 90, 70, 85]
ideal_values = [90, 90, 95, 100, 85, 95]
N = len(categories)
angles = np.linspace(0, 2 * np.pi, N, endpoint=False).tolist()  # Full circle divided
angles += angles[:1]  # Closing the loop

# Plot radar chart and calculate areas
household_polygon, ideal_polygon = plot_radar_chart(categories, household_values, ide
ideal_area, intersection_area, outside_area = calculate_areas(household_polygon, idea

print(f"Ideal Area: {ideal_area:.2f} square units")
print(f"Intersection Area: {intersection_area:.2f} square units")
print(f"Household Outside Ideal Area: {outside_area:.2f} square units")
```

Team 2024008 Radar
OpenAI ChatGPT (Feb 21, 2024, ChatGPT-4)
Query1: Come up with 6 very different household scenarios in Brisbane, Australia, include: - general status - free time - household type & area - disposable income - experience in handling pets - vet proximity.
Output: Used only as consultation

ChatGPT

Creating diverse household scenarios in Brisbane, Australia requires considering various factors like income levels, urban versus suburban settings, family compositions, and lifestyle choices. Here are six distinct scenarios:

1. The Young Professional

- **General Status**: Single, early 30s, working in a tech startup.
- **Free Time**: Enjoys exploring the city's nightlife, trying new restaurants, and weekend hiking.
- **Household Type & Area**: Apartment living in Fortitude Valley, an area known for its vibrant arts scene and nightlife.
- **Disposable Income**: Moderate, with a decent portion allocated to dining out, technology, and travel.
- **Experience in Handling Pets**: Minimal, as work and social life have been priorities.
- **Vet Proximity**: Close, with several options within a 10-minute drive, catering to the pet-friendly nature of the area.

2. The Suburban Family

- **General Status**: Married couple with two children, both parents working full-time.
- **Free Time**: Family-oriented activities, local sports clubs, and weekend barbecues.
- **Household Type & Area**: Detached home in The Gap, a family-friendly suburb with parks and schools.
- **Disposable Income**: Moderate to high, focused on home improvements, children's education, and family vacations.
- **Experience in Handling Pets**: Experienced, likely owning a dog or cat, with children involved in their care.
- **Vet Proximity**: Good, with a few veterinary clinics in the suburb or nearby areas.

3. The Retiree

- **General Status**: Widowed, in their late 70s, enjoying retirement.
- **Free Time**: Gardening, reading, participating in community groups, and visiting family.
- **Household Type & Area**: Smaller, manageable home or townhouse in a quiet part of Taringa.

- **Disposable Income**: Fixed, with income coming from pensions and savings, cautious with expenses.
- **Experience in Handling Pets**: High, possibly has had pets for many years, might currently have a smaller, low-maintenance pet.
- **Vet Proximity**: Accessible, with at least one veterinary clinic within a short driving distance.

4. The University Students

- **General Status**: Group of three university students sharing accommodation, studying full-time.
- **Free Time**: Socializing, part-time jobs, university clubs, and events.
- **Household Type & Area**: Shared rental property in St Lucia, close to the University of Queensland.
- **Disposable Income**: Low, mostly spent on rent, groceries, and occasional entertainment.
- **Experience in Handling Pets**: Varied, but the shared living situation might limit their ability to own pets.
- **Vet Proximity**: Moderate, with a few options but possibly less priority given to pet ownership due to lifestyle and rental agreements.

5. The Urban Couple

- **General Status**: DINK (Double Income, No Kids), mid-30s, both professionals working in the city.
- **Free Time**: Fine dining, arts and theatre, international travel.
- **Household Type & Area**: Modern apartment in New Farm, close to the Brisbane River, parks, and cultural hubs.
- **Disposable Income**: High, with significant spending on lifestyle, luxury items, and savings for future investments.
- **Experience in Handling Pets**: Moderate, may have a pet as a couple, focusing on pets that fit their lifestyle, like a cat or a small dog.
- **Vet Proximity**: Very good, with high-quality veterinary services nearby to cater to the upscale area.

6. The Rural-Edge Dweller

- **General Status**: Middle-aged couple with teenage children, one parent works in the city, and the other locally or from home.
- **Free Time**: Outdoor activities, gardening, DIY projects, and family outings.
- **Household Type & Area**: Large block in Samford Valley, offering a mix of rural and suburban living.
- **Disposable Income**: Moderate, with investments in property, vehicles for commuting, and outdoor equipment.
- **Experience in Handling Pets**: High, possibly owning multiple pets including dogs, cats, and perhaps horses or chickens.
- **Vet Proximity**: Limited, with fewer options requiring longer drives to access specialized or emergency veterinary care.

OpenAI ChatGPT (Feb 21, 2024, ChatGPT-4)
Query: Now develop a code to also output the scores using the formula: intersection_area / ideal_area + 1/5 * log(outside_area / ideal_area + 1/10) + 1/5
Output:

Certainly, here's a streamlined version of the code that calculates the areas and score for a single household against the ideal values without randomizing household values. The household values are directly provided:

python Copy code

```python
import numpy as np
from shapely.geometry import Polygon
import matplotlib.pyplot as plt

def values_to_polygon(values, angles):
    """Convert radar chart values and angles to a Shapely Polygon."""
    points = [[value * np.cos(angle), value * np.sin(angle)] for value, angle in zip(
    return Polygon(points)

def calculate_areas_and_score(household_values, ideal_values, angles):
    """Calculate areas and score for a single household against an ideal benchmark.""
    household_polygon = values_to_polygon(household_values, angles[:-1])
    ideal_polygon = values_to_polygon(ideal_values, angles[:-1])

    # Calculate areas
    ideal_area = ideal_polygon.area
    intersection_area = household_polygon.intersection(ideal_polygon).area
    outside_area = household_polygon.difference(ideal_polygon).area

    # Calculate the score using the provided formula
    score = (intersection_area / ideal_area) + (1/5) * np.log((outside_area / ideal_a

    return ideal_area, intersection_area, outside_area, score

# Provided household values
household_values = [35.56, 42.94, 8.24, 32.57, 92.29, 54.67]

# Provided ideal values for comparison
ideal_values = [44.35, 30.63, 73.01, 53.97, 27.08, 52.87]
```

```python
# Categories and angles for the radar chart
categories = ['Free Time (F)', 'Household Space (H)', 'Disposable Income (Di)',
              'Animal Experience (Ac)', 'Temperature Tolerance (T)', 'Distance from V
N = len(categories)
angles = np.linspace(0, 2 * np.pi, N, endpoint=False).tolist()
angles += angles[:1]  # Closing the loop

# Calculate areas and score for the household
ideal_area, intersection_area, outside_area, score = calculate_areas_and_score(househ

# Printing the results
print(f"Ideal Area: {ideal_area:.2f} square units")
print(f"Intersection Area: {intersection_area:.2f} square units")
print(f"Outside Ideal Area: {outside_area:.2f} square units")
print(f"Score: {score:.2f}")
```

This code directly uses the provided household values to calculate the intersection area, the area of the household outside the ideal, and computes the score based on the specified formula. It simplifies the approach by focusing on a singular household's comparison against the ideal values, offering precise insights into the household's readiness for pet ownership.

Team 2024008 Radar
OpenAI ChatGPT (Feb 21, 2024, ChatGPT-4)
Query: Now develop a code to run a simulation of 100 households with household values randomised around the base values by plus minus 10% of free time (35.56), household space (42.94), Disposable income (8.24), animal experience (32.57), temperature tolerance (92.29), distance from vet (54.67) for me and graph their score as well
Output:

Below is the complete Python code that incorporates everything discussed: generating random household values based on specified base values and a range of plus or minus 10%, comparing these households against provided ideal values, calculating the relevant areas using Shapely, determining scores for each household based on a specific formula, plotting these scores, and printing detailed results for analysis.

python

```python
import numpy as np
from shapely.geometry import Polygon
import matplotlib.pyplot as plt

def values_to_polygon(values, angles):
    """Convert radar chart values and angles to a Shapely Polygon."""
    points = [[value * np.cos(angle), value * np.sin(angle)] for value, angle in zip(
    return Polygon(points)

def calculate_areas_for_households(household_values_list, ideal_values, angles):
    """Calculate areas and scores for multiple households against an ideal benchmark.
    ideal_polygon = values_to_polygon(ideal_values, angles[:-1])
    ideal_area = ideal_polygon.area

    results = []
    for household_values in household_values_list:
        household_polygon = values_to_polygon(household_values, angles[:-1])
        intersection_area = household_polygon.intersection(ideal_polygon).area
        outside_area = household_polygon.difference(ideal_polygon).area

        # Calculate the score for each household using the provided formula
        score = (intersection_area / ideal_area) + (1/5) * np.log((outside_area / ide
        results.append((intersection_area, outside_area, score))

    return ideal_area, results
```

```
# Base values for generating random household values within a specified range
base_values = np.array([35.56, 42.94, 8.24, 32.57, 92.29, 54.67])
# Ideal values for comparison
ideal_values = [44.35, 30.63, 73.01, 53.97, 27.08, 52.87]

np.random.seed(42)  # Ensure reproducible random results
num_households = 100

# Generate household values list with random values within +/- 10% of the base values
household_values_list = [[np.random.uniform(low=value*0.9, high=value*1.1) for value

# Categories and angles for the radar chart
categories = ['Free Time (F)', 'Household Space (H)', 'Disposable Income (Di)',
              'Animal Experience (Ac)', 'Temperature Tolerance (T)', 'Distance from V
N = len(categories)
angles = np.linspace(0, 2 * np.pi, N, endpoint=False).tolist()
angles += angles[:1]  # Closing the loop

# Calculate areas and scores for all households
ideal_area, results = calculate_areas_for_households(household_values_list, ideal_val

# Plotting the scores for visualization
scores = [result[2] for result in results]
plt.figure(figsize=(12, 6))
plt.plot(range(1, num_households + 1), scores, marker='o', linestyle='-', color='blue
plt.title('Household Readiness Scores Compared to Ideal')
plt.xlabel('Household Number')
plt.ylabel('Score')
plt.grid(True)
plt.legend()
plt.show()

# Printing detailed results for the first 5 households
print(f"Ideal Area: {ideal_area:.2f} square units")
for i, (intersection_area, outside_area, score) in enumerate(results[:5], 1):
    print(f"\nHousehold {i}:")
    print(f"  - Intersection Area: {intersection_area:.2f} square units")
    print(f"  - Outside Ideal Area: {outside_area:.2f} square units")
    print(f"  - Score: {score:.2f}")
```

This code generates a line plot where each point represents a household's score calculated based on the intersection area, the area outside the ideal, and the ideal area, as per the specified formula. The x-axis represents the household number (for simplicity, numbered from 1 to 100), and the y-axis represents the calculated score. This visualization makes it easy to see how each household stacks up in terms of readiness and to identify which households are closer to or further from the ideal benchmark.