# IMMC 2017 The 3rd Annual International Mathematical Modelling Challenge (International) Summary Sheet

#### Abstract

Looking at how leaders worldwide deal with global issues in meetings like G20, it is not hard to tell the huge effort required to organize a large scale international meeting of importance. To enhance the efficiency of the meeting and the eventual outcome, one not only have to consider whether the environment and facilities are adequate, but often have to take into account the cost incurred, the accessibility of the location. Nevertheless, due to the common practice of voting for the venue or all participating countries taking turns to host such conferences, the impact brought about by jet lag, a sleeping disorder, is usually inevitable.

Hence, if we are to lower the effect of jet lag to ensure the participants' performance, it would require its participants from different parts of the world to compromise on the location of the conference to be held. The decision is affected by several parameters, including but not limited to the effect of jet lag which leads to a significant reduction in productivity, cost of flight, climate and political or socioeconomic situations, many of which are qualitative parameters that cannot be determined or compared with ease. In this investigation, we shall aim at quantifying some of these parameters and take minimizing the effect of jet lag on the participants as our main objective without neglecting the consideration of the cost incurred; however, for simplicity we will assume that issues regarding visa or politics and socioeconomic level have no influence on the decision.

In the first part, we proposed the *Jet Lag-Productivity model* based on literature review and regression, taking into account the effect of workload, drug use, difference in time zone, as well as the pre-existing conditions of the participant, in hopes of extending the *Productivity Model* widely used on biological investigations that measures one's circadian rhythm, sleep inertia and reservoir level. With this, we can evaluate the relationship between jet lag, which has time difference as its main cause, and the participants' productivity.

In the second part, an algorithm that computes the location in terms of the longitude that minimizes the average time zone difference has been derived. After that, with reference to distance between the chosen location and the participating countries using big circle distance, one may use the algorithm to find the point on the derived longitude that has shortest average distance from the designated locations, in-turn the cost minimizing location. We have looked into two principles that place emphasis on the highest average productivity and ensuring the level of participation, which is up to the committee to choose so as to cater their needs. It is thus used to test the two given scenarios.

Finally, a discussion on the models' strength and weakness and a sensitivity analysis are done to explore how changing the parameters, e.g. the use of drugs, may affect the final output. Meanwhile, we also attempted to propose a relatively simpler approach to model the relationship between one's productivity and jet lag based on historical data collected from the participants using analytic hierarchy process, which may be an alternative method than collecting experimental data to obtain the required parameters in the *Jet Lag-Productivity model*.

# A Mathematical Model on Minimizing the Effect of Jet Lag on the Productivity of Conference Participants

# 1. Overview

## 1.1. Problem Restatement

There are four parts of the problem that we aim to solve.

- I. To model the effect of jet lag, caused by time zone difference, on one's productivity
- II. To develop an algorithm to determine the optimum location for a meeting that minimizes the effect of jet lag
- III. To explore the effect of other factors, e.g. climate, cost, intensity of the conference, on the decision-making process
- IV. To apply the model on the given "Small Meeting" and the "Big Meeting" and other possible applications

# **1.2.** Primary Assumptions

- I. All participants can contribute equally to the conference
- II. In the selection process, no circumstantial factors will be taken into consideration, e.g. political and socioeconomic status, which is beyond our scope and focus of investigation on jet lag
- III. Minimizing the effect of jet lag is the first priority of the committee, whereby cost and other factors are of reference
- IV. All participants undergo the same recovery process from jet lag under the same light exposure with a fixed amount of rest time, i.e. 2 days, which would allow them to recover to the same extent
- V. All other costs, e.g. insurance, hotel booking, etc. are largely dependent on IMMC (the committee's) preference, and are hence not considered in the decision-making process

Terminology		
Time zone	The countries in the same time zone refer to those on the same longitude	
Productivity	Level of cognitive performance measured quantitatively	
Variables/ Functions		
x Time interval in minute as a person's productivity decreases		

# **1.3.** Primary Definitions

E(x)	Function that models the change in cognitive performance of a subject over time
P(x)	Function that models the change in <i>Productivity</i> of a subject over change in <i>time zone</i> (measured in radian)
x <sub>i</sub>	The shortest projected radian between a designated location and a chosen point along the equator
$d_i$	The shortest distance along the surface of Earth between a designated location and a chosen point

# 2. Jet Lag Influence Model

## 2.1. Assumptions

- 1. The primary model proposed is suited for an average adult based on extensive literature review *Justification: This is done to simplify the calculations and is made with various research*
- 2. No sleeping recovery is considered in *Productivity* model, which requires more literature support and we would assume all participants undergo a similar degree of recovery. *Justification: This is done to simplify the calculations*

## 2.2. Model

On account of the lack of actual data that measures the effect of jet lag on one's "productivity" which is an abstract concept, we identify the need to model an average person's *productivity* that changes over time, which we have adopted a quantitative approach in its measurement of the participant's cognitive response and performance based on literature review. The effect of jet lag on it will then be supplemented. With reference to the SAFTE model proposed in biological research that serves to model the cognitive performance of a person over time, there are three main factors that determine one's *productivity* over time *x* minutes.

1)  $R_x = R_{x-1} - K$ 

where  $R_x$  refers to the reservoir level, that is the energy level of a person that depletes with the coefficient *K* of 0.5 units where  $R_0 = 2880$  units;

2)  $C_T = cos(2\pi \frac{(x-p)}{24}) + \beta \cdot \frac{cos(4\pi (T-p-p'))}{24}$ 

where  $C_x$  refers to the circadian rhythm, in other words the sleeping pattern of a person, which p is the peak of the 24-hour rhythm and has a value of 18 (h), and p' is the peak of the 12-h rhythm, having a value of 3 (h), and  $\beta$  is a weighting factor with a value of 0.5. Here the variable x is represented in terms of hour;

3) 
$$I_x = -I_{max} e^{-(i - \frac{t}{-a_s C_x + f(R_c - R_x)})}$$

which  $I_x$  measures the sleep inertia, that is the state of reduced performance immediately after wakeup (i.e. within 2 hours).  $a_s = 0.55$  as a weighting factor by default, whereas f = 0.0026564. We would assume the maximum sleep inertia effect to be 5%, i.e.  $I_{max} = 0.05$ ;

By considering the weighting factors, the function of a subject's cognitive performance is combined as the following, :

$$E(x) = 100 \cdot \frac{R_x}{R_c} + (a_1 + a_2 \frac{R_c - R_x}{R_c}) \cdot C_x + I_x$$

whereby  $R_0$ , K and p, p' vary with the participants' pre-existing conditions and sleeping schedule respectively. Pre-existing conditions refer to one's age, body condition and fitness, as well as frequency of travel, altogether altering one's overall energy level and depletion rate. Such information, as well as the number of participants thus have to be requested from the participants. By calculating the mean using the number of participants, the standard deviation of each parameter can be calculated based on the real data, which can be applied to the formation of the E(x) by referring to the literature value corresponding to the participant's' condition.

To make it a more realistic model that caters specific situations, we then introduce different parameters into the function E(x). We then have

#### Jet lag Influence

There are two main factors that decide the overall impact including the direction of the flight and the time zone difference. The direction of travel may either enhance or lessen the effect of jet lag should the plane travel in the same direction as the rotation of Earth. As jet lag mainly interrupts one's circadian pattern, it changes the parameter p by

$$p_{East} = p' - 1.0 \times \frac{\Delta T}{24}$$
 for eastward travel  
 $p_{West} = p' + 1.5 \times \frac{\Delta T}{24}$  for westward travel

where as aforementioned, *T* is time interval *x* represented in hours and  $\Delta T$  refers to the time zone difference in hours between the destination city and the departure city or  $t_i$  representing the time zone difference between location  $p_i$  and a given point.

#### Use of Drug

The use of drugs refers to the use of substances that may boost or diminish one's cognitive performance, e.g. melatonin, alcohol, caffeine. Extending the use of the absorption factor of each substance, the relationship is

$$PB = 100 \times \frac{C}{C + K_{50}}$$

Where PB is a performance boost index we proposed to measure how a specific type of drug affects one's performance and C refers to the plasma concentration of the drug and  $K_{50}$  being the correspondent concentration that results in half of the maximum performance boost both obtained online from databases.

$$E(x) = 100 - (100 - E(x)_0) \frac{100 - PB}{100}$$

This hence illustrates how PB can be related to one's change in performance as we treat PB as a factor.

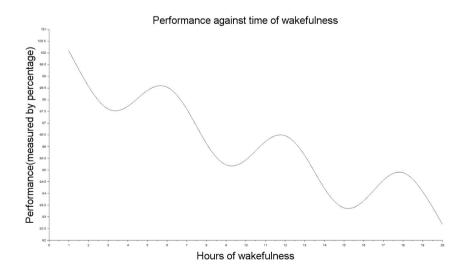
#### Workload

The workload of the conference refers to the intensity and intellectual demand of the tasks handled, which is proved in medical research to be amplifying the effect of jet lag. With reference to the SAFTE and DRDC model, it takes the form

$$W_{x} = W_{x-1} - (W_{d} \Delta x L R_{C}) / (100 + R_{x})$$
$$E(x) = E_{x^{0}} - \frac{W_{C} - W_{x}}{W_{c}}$$

where  $W_x$  is the reservoir level,  $W_c = 75$  units as the workload reservoir capacity,  $W_d = 1.14$  units/h as the depletion rate, Assume  $0 \le W_x \le W_c$ .  $W_r = 11$  units/h refers to the recovery rate while L is the rating of the task to be handled from zero to three to be determined by the committee. The depletion and recovery rates are proposed based on the *Fatigue and Risk Index* which gives the following graph for an average adult attending an "intensive three-day conference" with the regular intake of caffeine. The parameter L here is a scale from 0 to 3 that is intended to describe the difficulty of tasks. Since the conference is portrayed as "intensive", 3 is chosen to be an appropriate estimation of the task difficulty.

Cubic spline is then applied as the most suitable method of interpolation under this scenario to give a curve with smooth knots that allows the use of integration in the later procedure. Several piecewise functions linking the entire graph can thus be derived, which gives the following curve that illustrates an average adult's cognitive performance (*productivity*) with periodic features due to the presence of cosine function.



We would assume the conference takes place 2 hours after the participant wakes up, which, alike normal business or political conferences, last for 8 straight hours. Hereby, we have decided to refine the definition of the *productivity* of a person as the definitive integral between the performance of the person and the working hours of this person, where the performance of the person is measured as the percentage which has a maximum value of 100% because a person can at most perform fully to his/her potential. Productivity is defined as such because performance can be regarded as the rate of a person to finish different tasks. After taking the definite integral of performance and time, the total amount of work done in that period of time can be obtained which is the *productivity*. In this case, the definite integral between T=2 to T=9 will be calculated.

However, in some programming softwares, such the software SCILAB widely used in this paper, while the y-values and the slopes at each knots can be known, the piecewise function cannot be obtained directly. Therefore, the following equations should be used to used to arrive at the final piecewise functions. Let the piecewise function be q(x), then

$$q(x) = (1-t)y_1 + ty_2 + t(1-t)[a(1-t) + bt]$$

$$q(x) = \frac{a-b}{(x_2-x_1)^3}x^3 + \left[\frac{b-2a}{(x_2-x_1)^2} - \frac{3x_1(a-b)}{(x_2-x_1)^3}\right]x^2 + \left[\frac{3x_1^2(a-b)}{(x_2-x_1)^3} - \frac{2x_1(b-2a)}{(x_2-x_1)^2} + \frac{y_2-y_1+a}{x_2-x_1}\right]x + \left[\frac{x_1^2(b-2a)}{(x_2-x_1)^3} - \frac{x_1(y_2-y_1+a)}{(x_2-x_1)^3} - \frac{x_1(y_2-y_1+a)}{(x_2-x_1)^3} + y_1\right]x + \left[\frac{x_1^2(b-2a)}{(x_2-x_1)^3} - \frac{x_1(y_2-y_1+a)}{(x_2-x_1)^3} - \frac{x_1(y_2-y_1+a)}{(x_2-x_1)^3} + y_1\right]x + \left[\frac{x_1^2(b-2a)}{(x_2-x_1)^3} - \frac{x_1(y_2-y_1+a)}{(x_2-x_1)^3} - \frac{x_1(y_2-y_1+a)}{(x_2-x_1)^3} + y_1\right]x + \left[\frac{x_1^2(b-2a)}{(x_2-x_1)^3} - \frac{x_1(y_2-y_1+a)}{(x_2-x_1)^3} - \frac{x_1(y_2-y_1+a)}{(x_2-x_1)^3} - \frac{x_1(y_2-y_1+a)}{(x_2-x_1)^3} + y_1\right]x + \left[\frac{x_1^2(b-2a)}{(x_2-x_1)^3} - \frac{x_1(y_2-y_1+a)}{(x_2-x_1)^3} - \frac{x_1(y_2-y_1+a)}{(x_2-x_1)^3} - \frac{x_1(y_2-y_1+a)}{(x_2-x_1)^3} + y_1\right]x + \left[\frac{x_1^2(b-2a)}{(x_2-x_1)^3} - \frac{x_1(y_2-y_1+a)}{(x_2-x_1)^3} - \frac{x_1(y_2-x_1)}{(x_2-x_1)^3} - \frac{x$$

where,

$$q(x_{1}) = y_{1}$$

$$q(x_{2}) = y_{2}$$

$$q'(x_{1}) = k$$

$$q'(x_{2}) = k_{2}$$

$$t = \frac{x - x_{1}}{x_{2} - x_{1}}$$

$$a = k_{1}(x_{2} - x_{1}) - (y_{2} - y_{1})$$

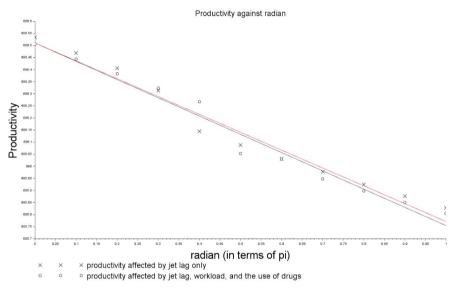
$$b = -k_{2}(x_{2} - x_{1}) + (y_{2} - y_{1})$$

By obtaining the area under the graph during the subject's working period, we have a quantifiable way to assess one's productivity with the jet lag factor introduced. The subject's productivity within the same working period will now change according to the effect of jet lag, whereby the two other factors, drug use and workload, may heighten or lower the effect of jet lag. Ultimately, the relationship between a subject's total productivity within a certain range of time and the extent of jet lag can be modelled with the available data points. By integrating the 2-9 hour section of the curve, the area under curve can be calculated.

In this paper, *time zone* are measured in a continuous manner instead of the conventional approach where countries, for instance China, may fall into only 1 time zone, despite its far-reaching terrain, so as to obtain more accurate results and more realistic predictions, having the same principle of calculating jet lag. Considering the fact that changing the latitude of a designated location will not result in any time

difference, we convert the 3D Earth into a 2D number line measured in radian with origin, the Prime Meridian as 0, and the right and left limit as  $\pi$  and  $-\pi$  respectively. Since a change in  $\pi$  radian results in a time difference of 12 hours, the time difference that corresponds to other values of radians can be calculated accordingly. Hence, distance travelled is essentially the radian travelled. This process is then repeated with different values of  $t_i$ , corresponding to the level of *productivity* under different time difference and the subsequently different severity of jet lag. We then derive the *Jet Lag-Productivity model* by applying several types of regression to find the one that best fits the data points, whereas we will discuss the following two in detail.

#### **Linear Regression**



As shown in the diagram, the crosses represent the productivity solely affected by different level of time difference, whereas the circles connote the productivity affected by jet lag, workload, as well as the use of drugs. Linear regression is first used as the most common method. The productivity curve considering the effects of all of the 3 parameters is thus

y = -0.7557x + 686.51

#### **Logistic Regression**

However, considering the pattern of the data points, sigmoid functions are also used in the form of logistic regression which are commonly applied in predicting growth, positive or negative, over time, which suits our objective . In particular, Gompertz function which has the general form as the following is chosen as it is a typical model in time series where growth is slowest in the beginning and the end of a time period as we believe that the effect of jet lag would be insignificant for  $t_i < 2$ , while for large values of  $t_i$ , the difference in incurred impact isn't as dramatic for the change already requires much effort to adjust the circadian rhythm.

$$y(t) = ae^{-be^{-ct}}$$

As a result, logistic regression is carried out and the following graph is obtained. However, since the entire curve is tilted, the following rotational matrix is multiplied to the original Gompertz Function.

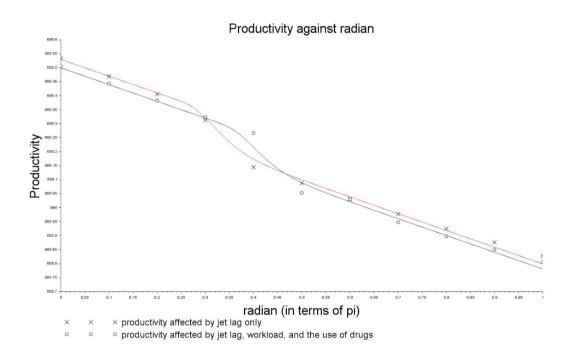
$$\begin{bmatrix} \cos\theta & -\sin\theta\\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x\\ y \end{bmatrix} = \begin{bmatrix} x'\\ y' \end{bmatrix}$$

$$y' = x \sin\theta + y \cos\theta = x \sin\theta + y(x) \cos\theta$$

$$y' = x \sin\theta + ae^{-be^{-cx}} \cos\theta$$

After manual fitting, the productivity function p(x) can be obtained as the following

$$p(x) = (x - m) \sin \theta + a e^{-be^{-c(x - m)}} \cos \theta + n$$



With the productivity curve considering all three factors with the parameters suiting that of an average adult being

$$p(x) = sin(-2.5)(x - 0.4) + cos(-2.5)(0.15)(e^{-e^{-25(x-0.4)}}) + 686.26$$

However, the derivation above is only one illustrative example of our model for eastward flights. Since the flight can travel in both the westward and eastward direction, the westward direction should also be taken into account, which can be done in a similar manner. After using both regression methods, logistic regression is chosen both from an analytical and an empirical perspective. Whereas the graph should never approach zero considering a person's productivity would never decrease to zero until reaching one's biological limit of five days, it is worth noting that a smallest average distance may not necessarily give a lowest average impact. Most people can adjust rapidly to a one or two hour time zone change. Three or more may cause more noticeable symptoms of jet lag. Hence, a linear relationship may not be realistic.

<u>Analytical Justification by  $R^2$ </u>

$$R^2 = 1 - \frac{SS_{res}}{SS_{tot}}$$

where  $f_i$  is the modelled value for each  $x_i$ 

$$SS_{tot} = \sum_{i} (y_i - \overline{y})^2$$
$$SS_{res} = \sum_{i} (y_i - f_i)^2$$

Having in mind that the closer the value of  $R^2$  is to 1, the better the curve predicts the trend. It can be readily found that the linear regression of the productivity curve of all three factors brought into consideration has a  $R^2 = 0.97418$  while the counterpart in the logistic regressions yields a  $R^2 = 0.99236$ . Thus we have adopted the logistic regression curve for the *Jet Lag-Productivity Model*.

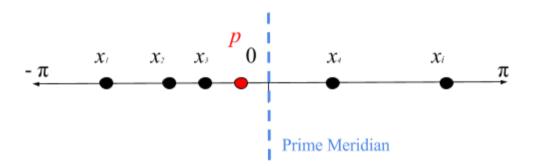
## 3. Location Model

## 3.1. Additional assumptions

- Unlike the conventional definition of time zone that takes in political and geographical considerations, we define *time zone* of a place by its corresponding longitude. Locations lying on the same longitude will lie in the same time zone. While the time zone difference of two locations is directly proportional to the difference in the two's longitudes. *Justification: This is done to simplify the calculations and is made with literature support.*
- 2. The path travelled is the shortest line along Earth's surface *Justification: This is done to simplify the calculations.*
- The calculated path has an error within ±0.5 % Justification: This is done to compensate for the fact that the Earth is not entirely spherical which is supported by literature.
- 4. All flights are booked in the same season and are direct flights, so as to avoid having cumbersome calculation for the committee as having connecting would dramatically complicate one's sleeping schedule with respect to the length of stay, where the distance-productivity relationship will not have an obvious trend *Justification: This is done to simplify the calculations*

## 3.2. Model

To minimize the effect of jet lag, we first consider the longitude of the countries in concern, as the differences in time zones are the main cause of jet lag. In order to single out the effect of latitude, we consider the difference in longitude of the locations in radians, instead of the horizontal component of the distance which is affected by the respective latitude, and then align the countries on a straight horizontal line  $L_1$ , ranging from 0 at the prime meridian to  $+\pi$  to its east of and  $-\pi$  to its west. Each country correspond to a point on the line, neglecting the differences in latitude. We have divided the line into 1000 sections to obtain the optimal longitude with high accuracy. The number of intervals can be changed in case the user wants an even higher accuracy.



Two cases have then to be considered to obtain the required longitude:

## **Case 1: Maximizing Overall Productivity To find** $max \sum P$

Considering a variable section  $\varepsilon$  of the line, we calculate the total productivity of people by summing up the productivity of participants from each country, which is achieved by fitting the difference between the point representing each country and  $\varepsilon$  to the productivity curve obtained in **2.2**. Similarly, the total productivity of participants on different sections of the line L<sub>1</sub> can be calculated. After the total productivity for all 1000 sections are determined, the section with the largest total productivity is chosen.

**Case 1** : For  $|p - x_i| < \pi$ ,  $p_{West}$  should be applied when  $p - x_i > 0$   $p_{East}$  should be applied when  $p - x_i < 0$ **Case 2** : For  $|p - x_i| > \pi$ ,

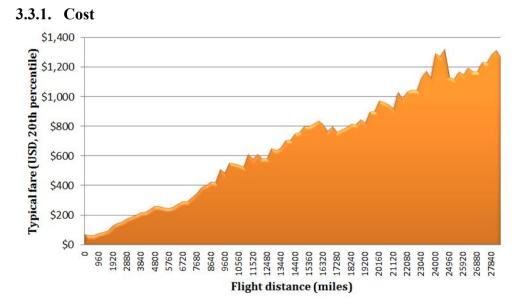
**Case 2**: For  $|p - x_i| > n$ ,  $p_{East}$  should be applied when  $p - x_i > 0$  $p_{West}$  should be applied when  $p - x_i < 0$  As no flight will fly a distance/radian difference larger than  $\pi$ , the flight will always choose the shortest route. A range of cities with the highest overall productivities will be chosen and shortlisted to be assessed using other criteria.

## Case 2: Prevention of extreme cases max (P<sub>min</sub>)

#### Method 2: Maximizing the Minimum Productivity

Similar to case 1, the productivity of each participant from each country can be determined by fitting the difference between the point representing each country and  $\varepsilon$ . Yet, the total productivity is no longer worthwhile, instead the minimum productivity is in concern. The section on L<sub>1</sub> with the highest minimum productivity is chosen out of the 1000 intervals, in order to ensure that all participants are able to have approximately contribution to the meeting.

The algorithm can thus compute a list of locations and rank them according to the two objectives above.

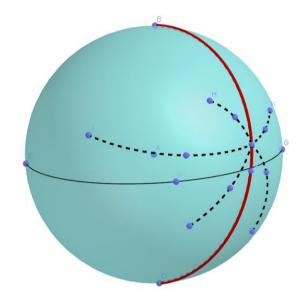


## **3.3.** Choosing the Optimal location

For the ease of investigation, we may first assume a positive linear relationship between flight distance and cost. A linear relationship would be  $Fare = \$50 + \$0.11 \cdot D_i$ .  $D_i$  varies by airline, e.g. if the committee chooses Turkish Airlines,  $D_i$  would have a value of 11.6. Hence, on the same longitude, all countries can be considered, while it is reasonable to assume that by finding the point where  $min \sum_{i=1}^{n} d_i$  is achieved, the cost would be lowest.

After the preferred longitude is chosen, no matter under case 1 or 2 with different objectives, the latitude of the optimum location is determined by comparing the great circle distance, which is the shortest distance between two points along the surface of a sphere, between the participating countries and the

location of meeting. The latitude contributing to the minimum total great circle distance is obtained by adding the great circle distance  $d_i$  between all participating countries and the location of interest. Minimizing the great circle distance also implies the reduction in cost of flight, as distance and cost are linearly related. Furthermore, a shorter great circle distance means a reduced effect of photoperiod, which is beneficial to the participants as differences in photoperiod is also a factor of jet lag. Similar to the determination of the longitude, from part **3.2**. A longitude  $L_2$  is chosen which ranges from 0 to  $-\frac{\pi}{2}$  to its North.



According to Great-circle distance and spherical laws of cosines, the distance between two points along the surface of the Earth can be calculated given their respective latitudes and longitudes which can be calculated as follows.

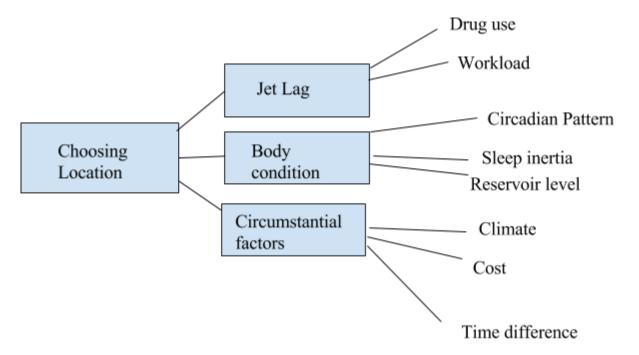
Given two points A and B on Earth, with their latitudes and longitudes being  $A(a_1, b_1)$  and  $B(a_2, b_2)$ , then

$$d = r \times \Delta \sigma$$

where *r* is the radius of the Earth and  $\Delta \sigma = \arccos \left[ (\sin a_1)(\sin a_2) + (\cos a_1)(\cos a_2)(\cos |b_1 - b_2|) \right]$ Since *r* is fixed, then minimizing *d*, in other words, is to minimize  $\Delta \sigma$ , whereby the error of  $\pm 0.5$  % approximately covers the size of a city as we only take one specific value for the latitude and longitude of a city.

## 3.4. Questionnaire model

Owing to the lack of real life data, part **3.4.** Only serves as an alternative approach for general decision making, if the committee is to consider more factors like the climate, other expenses, et cera. First, the committee have to collect the rating of the participants of several certain conferences on the importance



of different factors affecting one's choice of conference location. There are various factors to determining an optimal location, which an analytic hierarchy relationship diagram can be drawn.

Which by applying the analytic hierarchy process, a relative score  $a_{pq}$  (For the score table, please refer to appendix) of 1-9 can be given to the parameters above to represent it importance, while a pairwise comparison matrix A is constructed. The matrix A is a m×m real matrix, where m is the number of evaluation criteria considered, in this case m=9. Each entry  $a_{pq}$  of the matrix A represents the importance of the  $p^{th}$  criterion relative to the  $q^{th}$  criterion. If  $a_{pq} > 1$ , then the  $p^{th}$  criterion is more important than the  $q^{th}$  criterion, while if  $a_{pq} < 1$ , then the  $p^{th}$  criterion is less important than the  $q^{th}$  criterion.

$$\overline{a_{pq}}$$
 can then be computed as  $\frac{a_{pq}}{\sum\limits_{l=1}^{m} a_{lq}}$ 

While the weight vector is then  $w_j = \frac{\sum_{l=1}^{m} \overline{a_j l}}{m}$ , which can tell the weight of a particular factor in the decision-making process.

With the weight for each factor derived, the following matrix is computed

$$(X'_{1}, X'_{2}, \dots, X'_{N}) = \begin{array}{cccc} x'_{1}(1) & x'_{2}(1) & \cdots & x'_{n}(1) \\ x'_{1}(2) & x'_{2}(2) & \cdots & x'_{n}(2) \\ \vdots & \vdots & & \vdots \\ x'_{1}(k) & x'_{2}(k) & \cdots & x'_{n}(k) \end{array}$$

Where  $X'_i = (x'_1(1), x'_1(2), ..., x'_1(m)^{weight}, i = 1, 2, ..., n$  and n refers to the number of locations assessed. After adding in the weight factor and standardizing the values using  $X_i(j) = \frac{X_i(j) - X_i(j)_{min}}{X_i(j)_{max} - X_i(j)_{min}} \times 100\%$  for positive index/ score

$$X_{i}(j) = \frac{X_{i}(j)_{max} - X_{i}(j)}{X_{i}(j)_{max} - X_{i}(j)_{min}} \times 100\% for negative index/ score$$

The standard scores of each factor of each location can then be compared for future reference.

# 4. Application on the given scenario

## 4.1. Assumptions

- 1. The primary objective of IMMC is to have the highest average productivity
- 2. The participants are all average adults to be participating in an "intensive" meeting, where L = 3

# 4.2. Small Meeting

# **Data and Parameters**

	Monterey CA, USA	Zutphen, NLD	Melbourne , AUS	Shanghai, CHN	Hong Kong, CHN	Moscow, RUS
Longitude	121.8947 ° <i>W</i>	6.1961 °E	144.9631 °E	121.47 °E	114.11 °E	37.617 °E
Latitude	36.6002°N	52.1427 °N	37.8136 ° <i>S</i>	31.230 °N	22.396 °N	55.756 °N

## Results

$\sum p$	Location (longitude, latitude)	City	Final Decision
4115.6908	$(52.162162^{\circ}N, 66.126126^{\circ}W)$	Wabush, Canada	
4115.5978	(5.3153153 °N, 58.198198°E)	N/A	
4115.6018	(39.189189 °N , 101.44144 °E)	JinChang, China	1

As seen from the above table, the three places with the highest sum of productivity are shortlisted, which the list is not limited to three venues but can be customized based on the needs of the committee. However, as mentioned above, we also have to take climate and photoperiod into account, which both are of crucial role to the one's productivity under the influence of jet lag.

City	Average temperature (°C)	Latitude
Monterey, RUS	17	36.6002°N
Zutphen, NLD	20	52.1427 °N

Melbourne, AUS	11	37.8136 ° <i>S</i>
Shanghai, CHN	26	31.230 °N
Hong Kong, CHN	23	22.396 °N
Moscow, RUS	23	55.756 °N
Mean	20	26.719 °N
Median	21.5	33.915 °N
Wabush, CAN	13	52.162162°N
Jinchang, CHN	18.5	39.189189 °N

As a general principle for the committee to choose from the list or as a rule of thumb, the temperature of the venue should not be of great difference to the original temperature, which a significant discrepancy will result in an enhanced impact of jet lag, i.e. larger value of *p*. Meanwhile, photoperiods also have to be considered as it is another factor that affects the severity of jet lag. As the location found in the *Location Model* may not necessarily be the one that is with the lowest total latitude difference, it requires further evaluation as the larger photoperiod difference, which varies along the same latitude, the longer it takes to adjust one's circadian pattern. Hence, with this principle, the suggested location is Jinchang, China.

## Alternative:

Using the same rationale, the case of preventing extremely low productivities and prompting equal contribution can be done. The place is found to be Bannikoppa, India and has the latitude and longitude (15.405405°N, 75.495495°E)

## 4.3. Big Meeting

#### **Data and Parameters**

	Boston MA, USA (2)	Utrecht, NLD	Melbourne , AUS	Beijing, CHN	Hong Kong, CHN (2)	Moscow, RUS
Longitude	71.059 ° <i>W</i>	5.1214 °E	144.96 °E	116.41 °E	114.11 °E	37.617 °E
Latitude	42.360°N	52.091 °N	37.814 ° <i>S</i>	39.904 °N	22.396 °N	55.756 °N

	Warsaw, POL	Singapore, SGP	Copenhagen, DNK
Longitude	21.012 °E	103.82 °E	12.568 °E
Latitude	52.230°N	1.3521 °N	55.676 °S

## Results

$\sum p$	Location (longitude, latitude)	City	
7546.5205	$(24.414414^{\circ}N, 109.00901^{\circ}E)$	Liucheng, China	
7546.0289	(35.765766 °N, 118.37838°E)	Yishui, China	✓
7546.4091	(35.945946 °N , 112.61261 °E)	Yangcheng, China	

city	temperature	latitude
Boston	-1.67	42.360°N
Boston	-1.67	42.360°N
Singapore	26	1.3521 °N
Beijing	-5.5	39.904 °N
Hong Kong	16.5	22.396 °N
Hong Kong	16.5	22.396 °N
Moscow	-9	55.756 °N
Utrecht	2	52.091 °N
Warsaw	-2	52.230°N
Copenhagen	0	55.676 ° <i>S</i>
Melbourne	21	37.814 ° <i>S</i>
Mean	5.65	21.578°N
Median	0	39.904 °N

Liucheng	21.7	24.414414°N
Yishui	2	35.765766 °N
Yangcheng	0	35.945946 °N

From the above table, it is evident that after comparing the difference in temperature and latitude, that is the climate and the photoperiod, regions around Yishui should be the most suitable place for holding the meeting.

Alternative: Approximate equal contribution

Using a similar approach, the region found has a coordinate of (36.6667°N, 109.00901°E) which is closest to Yanan, China.

# 5. Sensitivity and Evaluation

The models used are based on parameters and the characteristics of an average human being, but in reality one may have to consider other factors that correspond to the conditions of every single human being differ in terms of age, gender, health and many other factors that may affect the effect of jet lag on a person's productivity. This may be achieved by researching and collecting response from the participants, which is possible to be simulated using normal distribution as discussed. Extensive research in this field is required to obtain an accurate representation of how changing these factors would affect our model. For instance, it is obvious that jet lag would have a larger impact on a person with less ideal health conditions, than an average healthy adult, but we are then faced with two problems. Firstly, data such as health conditions on the overall outcome of our model we must first identify a way to transform the qualitative data at hand into quantitative data. Even with the quantitative data collected, the way in which it should be applied to the model can only be determined through experimental data. The sensitivity of a model is determined by a weighing factor that is applied on the parameters, but in order to find the value of this weighing factor, primary research must be conducted in order to observe the effect this has on our model.

# 6. Discussion (Strength and Weakness)

## Strength

The *Productivity* model proposed considers various parameters that cater the needs of different conferences and the body condition of individuals, which is wholistic and mostly research-based that adds on to the model's reliability. It can also provide a relatively accurate prediction of one's performance under the influence of jet lag, that can be used separately.

Furthermore, the *Jet Lag-Productivity model* which is built based on the first model effectively relates the travelling distance to one's productivity based on actual data, which would presumably reflect the

real situation realistically with the various regression approaches tested both empirically and mathematically.

Nevertheless, in the calculation of the optimal location, the model is sophisticated with a small interval, which allows a considerably accurate calculation compared to the conventional approach that utilizes geographical time zones. The two-step approach also allows a list of locations that may not be close to one another as we first locate various longitudes that maximizes the productivity of the participants, which provides the committee with more diverse options.

Finally, the general rule of thumb and the AHP method proposed can allow the committees to have a easier guideline to assist their decision-making process on different qualitative and empirical parameters, which overall makes the model a more comprehensive and user-friendly tool.

#### Weakness

We have not considered the factor of climate and date in a quantitative approach due to the time constraint, whereby the discrepancy in climate, price level, may be explored to improve the idea of weighting of different factors when it comes to choosing a venue for an international conference. Some of the parameters chosen have to be derived experimentally, which may require extensive research or acquiring such resources online. Some important assumptions made are that the rest and recovery of the participants are neglected, while the other expenses incurred are also not of concern and no transfer flights are considered. They are mainly done to reduce the complicatedness of the calculation, while the effect of rest may be looked into in the future as it certainly plays an important role in affecting a person's productivity.

## 7. Conclusion

To model how jet lag and other factors influence one's productivity, we have proposed the *Productivity model* and extending it with several parameters, we constructed the *performance curve* which shows the relationship between cognitive performance and the time of wakefulness. After that, by adding the effects of jet lag, workload, and the use of drugs, a productivity curve can then be obtained by integrating the *performance curve*. Two types of regressions, namely the linear regression and the logistic regression are done to compare and contrast the results so as to achieve the best-fitting result. After acquiring the *productivity function*, we constructed the *location model* to accurately predict the optimal place for holding the meeting considering the overall productivity or to maximize the minimum productivity so as to ensure an approximate contribution from different individuals. Having fixed the longitude, latitude can also be obtained, by utilizing the great-circle distance formula, under the principle to minimize the total distance travelled. Having the latitude and longitude,

The committee can then choose one objective that they deem appropriate, while based on research and the response collected based on the questionnaire model and other factors like those explored in this paper, temperature and photoperiod, they can further evaluate the importance of each factor that can hardly be quantified. All in all, we hope this may serve as a concise and accurate indicator of the

optimum location for a conference to facilitate organizing committees' complicated process of decision-making.

# Appendix

Derivation

$$\frac{dA_1}{dt} = -k_a A_1$$

$$\frac{dC}{dt} = k_a \frac{A_1}{V_c} - \frac{CL}{V_c}C$$

 $A_1$  is the amount of drug in mg in the absorption from an initial dose, and C is the concentration of drug in plasma in mg/L, CL is the plasma clearance in L/min, Vc is the volume of distribution in L

The parameters for these equations were estimated based on the half-lives and time to peak concentration for each of the drugs. The half-lives were 5.3 h, 11.0 h, and 12.0 h and the time to peak concentrations were 1 h, 4 h, and 4 hfor caffeine, modafinil, and dextroamphetamine respectively. This resulted in ka values of 0.064, 0.013, and 0.013 and CL values of 0.083, 0.067, and 0.058, and Vc values of 38.6, 66.8, 460, respectively.

*Fatigue and Risk Index:* States that a break of thirty minutes length is sufficient for a complete recovery after six hours of work. The parameters were then optimized to fit a number of published data sets for performance on cognitive tasks with various levels of difficulties.

Relative scores:

Obviously, $a_{ij} = 1$ for all <i>j</i> . The relative importance between two criteria is measured according to a numerical scale from 1 to 9, as shown in Table 1, where it is assumed that the <i>j</i> th criterion is equally	
or more important than the kth criterion. The phrases in the "Interpretation" column of Table 1 are only suggestive, and may be used to translate the decision maker's qualitative evaluations of the	
relative importance between two criteria into numbers. It is also possible to assign intermediate values which do not correspond to a precise interpretation. The values in the matrix $\mathbf{A}$ are by construction pairwise consistent, see (1). On the other hand, the ratings may in general show slight	
inconsistencies. However these do not cause serious difficulties for the AHP.	

Value of a <sub>ik</sub>	Interpretation
1	j and k are equally important
3	j is slightly more important than k
5	j is more important than $k$
7	j is strongly more important than k
9	j is absolutely more important than $k$

Table 1. Table of relative scores.

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## Programme

1. Performance curve

clear;

clf;

xdata=[1:20];

```
ydata=[100.0963824,98.582682,97.60716146,97.72024108,98.39118752,98.53574592,97.6024784 4,96.13140496,95.22078125,95.42972424,96.21402929,96.46481544,95.61682159,94.21807221,9 3.38580729,93.68732141,94.56492875,94.885625,94.07132935,92.67954748];
```

d=splin(xdata,ydata,"natural");

xlist=min(xdata):0.001:max(xdata); //alternative of linespace

```
[y_cs,y1_cs]=interp(xlist,xdata,ydata,d);
```

plot2d(xlist,y\_cs);

xtitle("Performance against time of wakefulness", "Hours of wakefulness", "Performance(measured by percentage)")

//y\_cs is the point
//y1\_cs is the slope of the point

//Find the piecewise equation
n=1:8

k(n)=d'(n)x(n)=xdata'(n)y(n)=ydata'(n)

2. Linear Regression
clear;
//fit the circles (all effect, use the black line)
function [x]=linear1(k);
 x=-0.7557\*k+686.51;
endfunction

```
y=[];
for k1=0:0.01:1;
y=[y, linear1(k1)]
end
```

```
//fit the cross (the jet lag line, the red line)
function [x]=linear2(k);
    x=-0.7388*k+686.51;
endfunction
```

```
y2=[];
for k5=0:0.01:1;
y2=[y2, linear2(k5)]
end
```

```
k2=[0:0.01:1]';
plot2d(k2,[y',y2'],[1,5],leg="linear regression for jet lag cosidering use of drugs and
wokload@linear regression for jet lag") //black line is used to fit the circles
//red is the line to fit the cross
k3=[0:0.1:1]';
```

```
p_jet=[686.5338
686.4687
686.4054
686.3122
686.1439
686.0873
686.0305
685.9769
685.9252
685.8755
```

685.828];

p\_all=[686.5055 686.4432 686.3823 686.3233 686.2668 686.0529 686.0295 685.948 685.8983 685.8508 685.8049]; plot2d(k3 [p, jet p)

plot2d(k3,[p\_jet,p\_all],[-2,-9],leg="productivity affected by jet lag only@productivity affected by jet lag, workload, and the use of drugs") //circle is only jet lag xtitle("Productivity against radian", "radian (in terms of pi)", "Productivity")

#### 3. Logistic Regression

clear; //fit the circles (all effect, use the black line) m1=0.4; //horizontal translation n1=686.26; //vertical translation t1=-2.5; //rotation a1=0.15; //length of the "s" b1=1; c1=25; function [x]=logistic1(k); x=(k-m1)\*sin(t1)+cos(t1)\*a1\*(%e^(-b1\*%e^(-c1\*(k-m1))))+n1; endfunction

```
y=[];
for k1=0:0.01:1;
y=[y, logistic1(k1)]
end
```

```
//fit the cross (the jet lag line, the red line)
m2=0.3; //horizontal translation
n2=686.35; //vertical translation
t2=-2.5; //rotation
a2=0.165; //length of the "s"
b2=1.5;
c2=25;
```

```
y2=[];
for k5=0:0.01:1;
y2=[y2, logistic2(k5)]
end
```

```
k2=[0:0.01:1]';
```

```
plot2d(k2,[y',y2'],[1,5],leg="Gompertz function for jet lag cosidering use of drugs and
wokload@Gompertz function for jet lag") //black line is used to fit the circles
//red is the line to fit the cross
k3=[0:0.1:1]';
```

p jet= 686.5338 686.4687 686.4054 686.3122 686.1439 686.0873 686.0305 685.9769 685.9252 685.8755 685.828]; p all=[686.5055 686.4432 686.3823 686.3233 686.2668 686.0529 686.0295 685.948 685.8983 685.8508 685.8049];

plot2d(k3,[p\_jet,p\_all],[-2,-9],leg="productivity affected by jet lag only@productivity affected by
jet lag, workload, and the use of drugs") //circle is only jet lag
xtitle("Productivity against radian", "radian (in terms of pi)", "Productivity")

## 4. Location model on the small meeting (case 1)

```
clear;
//input the longitude of different points in degrees to get their respective positions on 2D
l=[-121.8947;6.1961;144.9631;121.4737;114.1095;37.6173];
p=(%pi/180)*l; //p is the position of the points on 2D
```

```
//find the point such that the sum of the y-values is the minimum
//k is a random point on 2D
k1=linspace(-%pi,%pi,1000)'; //change the intervals
distance2=[];
for x1=1:length(k1);
  distance2=abs([distance2, p-k1(x1)]);
end
m=length(distance2(:,1));
n=length(distance2(1,:));
distance=[distance2];
for i=1:m;
  for j=1:n;
  if distance(i,j)>%pi;
     then distance(i,j)=distance(i,j)-%pi;
  end
  end
end
function [x]=productivity(a);
  x=(a-0.4)*sin(-2.5)+cos(-2.5)*0.15*(%e^{-(-%e^{-25*(a-0.4))})+686.26;
endfunction
i=[]; //i is the matrix of severity
for x2=1:length(k1);
  i=[i, productivity(distance(:,x2))];
```

```
end
```

s=sum(i,1) //1 represents the sum in one column & s represents the sum of impact max(s)

```
//change the k thing
o2=k1(317)*180/%pi; //find the corresponding column + o2 represents the longtitude of the point
//Finish finding the longtitude of the point
```

//Find the altitude of the point by considering the shortest distance along the surface //Input the altitude and longtitude of the different cities altdegree=[36.6002;52.1427;-37.8136;31.2304;22.3964;55.7558]; longdegree=[-121.8947;6.1961;144.9631;121.4737;114.1095;37.6173]; alt=altdegree\*%pi/180; long=longdegree\*%pi/180; k2=linspace(-%pi/2,%pi/2,1000)'; deltalong=abs(long-k1(317)); //change the k thing

```
//Find the first portion
z1=[];
for x3=1:length(k2);
    z1=[z1, alt*k2(x3)];
end
```

```
//Find the second portion
z2_1=[];
for x4=1:length(k2);
    z2_1=[z2_1, alt*cos(k2(x4))];
end
```

z2=[]; for x5=1:length(k2); z2=[z2, z2\_1(:,x5).\*cos(deltalong)]; end

sigma=acos(z1+z2); totald=sum(sigma,1) o1=k2(790)\*180/%pi; disp([o1,o2])

## 5. Location model on small meeting (case 2)

clear;

//input the longitude of different points in degrees to get their respective positions on 2D l=[-121.8947;6.1961;144.9631;121.4737;114.1095;37.6173];p=(%pi/180)\*l; //p is the position of the points on 2D

//find the point such that the sum of the y-values is the minimum
//k is a random point on 2D
k1=linspace(-%pi,%pi,1000)'; //change the intervals
distance2=[];
for x1=1:length(k1);

```
distance2=abs([distance2, p-k1(x1)]);
end
```

```
m=length(distance2(:,1));
n=length(distance2(1,:));
```

```
distance=[distance2];
for i=1:m;
  for j=1:n;
    if distance(i,j)>%pi;
      then distance(i,j)=distance(i,j)-%pi;
    end
    end
end
```

```
function [x]=productivity(a);
x=(a-0.4)*sin(-2.5)+cos(-2.5)*0.15*(%e^(-%e^(-25*(a-0.4))))+686.26;
endfunction
```

```
i=[]; //i is the matrix of severity
for x2=1:length(k1);
    i=[i, productivity(distance(:,x2))];
end
```

```
m=[];
for x3=1:length(k1);
    m=[m, min(i(:,x3))];
end
//1 represents the sum in one column & s represents the sum of impact
o=max(m)
```

//change the k thing
o2=k1(710)\*180/%pi; //find the corresponding column + o2 represents the longtitude of the point
//Finish finding the longtitude of the point

```
//Find the altitude of the point by considering the shortest distance along the surface
//Input the altitude and longtitude of the different cities
altdegree=[36.6002;52.1427;-37.8136;31.2304;22.3964;55.7558];
longdegree=[-121.8947;6.1961;144.9631;121.4737;114.1095;37.6173];
alt=altdegree*%pi/180;
long=longdegree*%pi/180;
k2=linspace(-%pi/2,%pi/2,1000)';
```

```
deltalong=abs(long-k1(710)); //change the k thing
```

```
//Find the first portion
z1=[];
for x3=1:length(k2);
z1=[z1, alt*k2(x3)];
end
```

//Find the second portion
z2\_1=[];
for x4=1:length(k2);
 z2\_1=[z2\_1, alt\*cos(k2(x4))];
end

```
z2=[];
for x5=1:length(k2);
z2=[z2, z2_1(:,x5).*cos(deltalong)];
end
```

```
sigma=acos(z1+z2);
totald=sum(sigma,1)
o1=k2(586)*180/%pi;
disp([o1,o2])
```

## 6. Location model on the big meeting (case 1)

```
clear;
//input the longitude of different points in degrees to get their respective positions on 2D
l=[-71.0589;-71.0589;103.8198;116.4074;114.1095;114.1095;37.6173;5.1214;21.0122;12.5683;144
.9631];
p=(%pi/180)*l; //p is the position of the points on 2D
```

```
//find the point such that the sum of the y-values is the minimum
//k is a random point on 2D
k1=linspace(-%pi,%pi,1000)'; //change the intervals
distance2=[];
for x1=1:length(k1);
    distance2=abs([distance2, p-k1(x1)]);
end
```

```
m=length(distance2(:,1));
n=length(distance2(1,:));
```

```
distance=[distance2];
for i=1:m;
for j=1:n;
if distance(i,j)>%pi;
then distance(i,j)=distance(i,j)-%pi;
end
end
end
```

```
function [x]=productivity(a);
x=(a-0.4)*sin(-2.5)+cos(-2.5)*0.15*(%e^(-%e^(-25*(a-0.4))))+686.26;
endfunction
```

```
i=[]; //i is the matrix of severity
for x2=1:length(k1);
    i=[i, productivity(distance(:,x2))];
end
```

```
s=sum(i,1) //1 represents the sum in one column & s represents the sum of impact max(s)
```

```
//change the k thing
o2=k1(813)*180/%pi; //find the corresponding column + o2 represents the longtitude of the point
//Finish finding the longtitude of the point
```

```
//Find the altitude of the point by considering the shortest distance along the surface
//Input the altitude and longtitude of the different cities
altdegree=[42.3601;42.3601;1.3521;39.9042;22.3964;22.3964;55.7558;52.0907;52.2297;55.6761;-
37.8136];
longdegree=[-71.0589;-71.0589;103.8198;116.4074;114.1095;114.1095;37.6173;5.1214;21.0122;1
2.5683;144.9631];
alt=altdegree*%pi/180;
long=longdegree*%pi/180;
k2=linspace(-%pi/2,%pi/2,1000)';
deltalong=abs(long-k1(813)); //change the k thing
```

```
//Find the first portion
z1=[];
for x3=1:length(k2);
    z1=[z1, alt*k2(x3)];
end
```

//Find the second portion
z2\_1=[];
for x4=1:length(k2);
 z2\_1=[z2\_1, alt\*cos(k2(x4))];
end

```
z2=[];
for x5=1:length(k2);
z2=[z2, z2_1(:,x5).*cos(deltalong)];
end
```

```
sigma=acos(z1+z2);
totald=sum(sigma,1)
o1=k2(700)*180/%pi;
disp([o1,o2])
```

## 7. Location model on the big meeting (case 2)

```
clear;
//input the longitude of different points in degrees to get their respective positions on 2D
l=[-71.0589;-71.0589;103.8198;116.4074;114.1095;114.1095;37.6173;5.1214;21.0122;12.5683;144
.9631];
p=(%pi/180)*l; //p is the position of the points on 2D
```

```
//find the point such that the sum of the y-values is the minimum
//k is a random point on 2D
k1=linspace(-%pi,%pi,1000)'; //change the intervals
distance2=[];
for x1=1:length(k1);
    distance2=abs([distance2, p-k1(x1)]);
end
```

```
m=length(distance2(:,1));
n=length(distance2(1,:));
distance=[distance2];
for i=1:m;
  for j=1:n;
    if distance(i,j)>%pi;
      then distance(i,j)=distance(i,j)-%pi;
    end
    end
```

end

```
function [x]=productivity(a);
x=(a-0.4)*sin(-2.5)+cos(-2.5)*0.15*(%e^(-%e^(-25*(a-0.4))))+686.26;
endfunction
```

```
i=[]; //i is the matrix of severity
for x2=1:length(k1);
    i=[i, productivity(distance(:,x2))];
end
```

```
s=sum(i,1) //1 represents the sum in one column & s represents the sum of impact max(s)
```

```
//change the k thing
o2=k1(813)*180/%pi; //find the corresponding column + o2 represents the longtitude of the point
//Finish finding the longtitude of the point
```

```
//Find the altitude of the point by considering the shortest distance along the surface
//Input the altitude and longtitude of the different cities
altdegree=[42.3601;42.3601;1.3521;39.9042;22.3964;22.3964;55.7558;52.0907;52.2297;55.6761;-
37.8136];
longdegree=[-71.0589;-71.0589;103.8198;116.4074;114.1095;114.1095;37.6173;5.1214;21.0122;1
2.5683;144.9631];
alt=altdegree*%pi/180;
long=longdegree*%pi/180;
k2=linspace(-%pi/2,%pi/2,1000)';
deltalong=abs(long-k1(813)); //change the k thing
```

```
//Find the first portion
z1=[];
for x3=1:length(k2);
z1=[z1, alt*k2(x3)];
end
```

```
//Find the second portion
z2_1=[];
for x4=1:length(k2);
    z2_1=[z2_1, alt*cos(k2(x4))];
end
```

z2=[];

for x5=1:length(k2); z2=[z2, z2\_1(:,x5).\*cos(deltalong)]; end

```
sigma=acos(z1+z2);
totald=sum(sigma,1)
o1=k2(700)*180/\%pi;
disp([o1,o2])
```